

Unit 3: Quadratic Equations

3.4 The Quadratic Formula

The Quadratic Formula: The roots of $ax^2 + bx + c = 0$ are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

* on formula sheet

ex. Solve using the quadratic formula:

$$3x^2 + 5x - 2 = 0$$

$$a = 3 \quad b = 5 \quad c = -2$$

$$x = \frac{-5 \pm \sqrt{5^2 - 4(3)(-2)}}{2(3)}$$

$$= \frac{-5 \pm \sqrt{25 + 24}}{6}$$

$$= \frac{-5 \pm 7}{6}$$

$$\frac{-5+7}{6} \text{ or } \frac{-5-7}{6}$$

$$x = \frac{1}{3} \text{ or } -2$$

$$9x^2 + 4 = -12x$$

$$9x^2 + 12x + 4 = 0$$

$$a = 9 \quad b = 12 \quad c = 4$$

$$x = \frac{-12 \pm \sqrt{12^2 - 4(9)(4)}}{2(9)}$$

$$= \frac{-12 \pm \sqrt{144 - 144}}{18}$$

$$= \frac{-12 \pm 0}{18}$$

$$= -12/18 = -2/3$$

Discriminant:

$$D = b^2 - 4ac$$

The value of the discriminant (D) determines the nature of the roots

$D > 0$ 2 distinct real roots

$D = 0$ 1 distinct real root (2 equal real roots)

$D < 0$ no real roots

ex. Use the discriminant to determine the nature of the roots for each quadratic. Verify by graphing.

$$3x^2 - 5x = -9$$

$$3x^2 - 5x + 9 = 0$$

$$D = b^2 - 4ac$$

$$= (-5)^2 - 4(3)(9)$$

$$= 25 - 108$$

$$= -83 < 0$$

$\therefore$ no real roots

$$4\left(\frac{x^2}{4}\right) = (3x - 9)^2$$

$$x^2 = 12x - 36$$

$$x^2 - 12x + 36 = 0$$

$$D = b^2 - 4ac$$

$$= (-12)^2 - 4(1)(36)$$

$$= 144 - 144$$

$$= 0$$

$\therefore$ 1 real root

ex. Solve using the quadratic formula. Answers in exact form and as decimals rounded to the nearest hundredth.

$$3x^2 - 8x + 2 = 0$$

$$a = 3 \quad b = -8 \quad c = 2$$

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(3)(2)}}{2(3)}$$

$$= \frac{8 \pm \sqrt{64 - 24}}{6}$$

$$= \frac{8 \pm 2\sqrt{10}}{6}$$

$$= \left[\frac{4 \pm \sqrt{10}}{3} \right] \approx \begin{matrix} 2.39 \\ 0.28 \end{matrix}$$

$$2x^2 + 3x = 1$$

$$2x^2 + 3x - 1 = 0$$

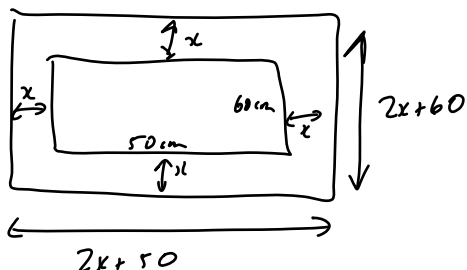
$$a = 2 \quad b = 3 \quad c = -1$$

$$x = \frac{-3 \pm \sqrt{3^2 - 4(2)(-1)}}{2(2)}$$

$$= \frac{-3 \pm \sqrt{9 + 8}}{4}$$

$$= \left[\frac{-3 \pm \sqrt{17}}{4} \right] \approx \begin{matrix} 0.28 \\ -1.78 \end{matrix}$$

ex. Leah wants to frame an oil original painted on canvas measuring 50 cm by 60 cm. Before framing, she places the painting on a rectangular mat so that a uniform strip of the mat shows on all sides of the painting. The area of the mat is twice the area of the painting. How wide is the strip of exposed mat showing on all sides of the painting, to the nearest tenth of a centimetre?



$$(2x + 50)(2x + 60) = 2(50)(60)$$

$$4x^2 + 120x + 100x + 3000 = 6000$$

$$\frac{4x^2 + 220x - 3000}{4} = \frac{0}{4}$$

$$x^2 + 55x - 750 = 0$$

$$x = \frac{-55 \pm \sqrt{55^2 - 4(1)(-750)}}{2(1)}$$

$$= \frac{-55 \pm \sqrt{6025}}{2}$$

$$\approx 11.3 \quad \text{or} \quad -66.3$$

$$\boxed{11.3 \text{ cm}}$$

Practice: pg. 254/ # 1cde, 3adf, 4de, 5, 7 (use a different algebraic method for each), 8, 10 – 13, 21, 22