PreCalculus 11 Date ________________

Unit 2: Quadratic Functions
2.4 Completing the Square

We convert a quadratic function from standard form $f(x) = ax^2 + bx + c$ to vertex form $f(x) = a(x-p)^2 + q$ using a process called **completing the square**

ex. Convert to vertex form: $y = x^2 + 8x + 10$

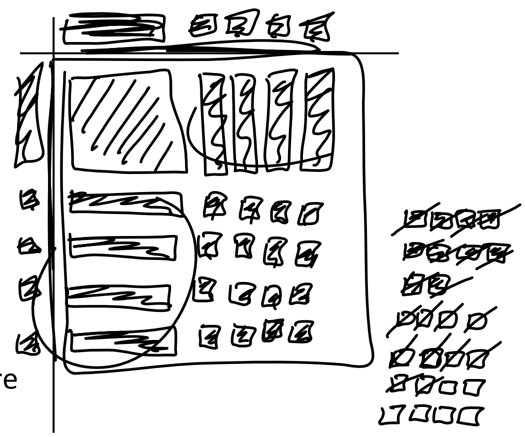
*recall algebra tile method & replicate algebraically

Step | Algebra tiles | Algebraic

1) Lay out trinomial
- separate constant

2) complete the square
- balance out the added values

3) factor the perfect square
- combine constants



$y = (x^2 + 8x + \underline{16}) + 10 - 16$

$y = \underbrace{(x^2 + 8x + 16)} - 6$

$y = (x+4)^2 - 6$

ex. convert each quadratic to vertex form:

$f(x) = x^2 - 2x + 3$

$= (x^2 - 2x + \underline{1}) + 3 - 1$

$\left(\frac{2}{2}\right)^2$

$\boxed{f(x) = (x-1)^2 + 2}$

$y = x^2 + 3x + 1$

$= (x^2 + 3x + \underline{\tfrac{9}{4}}) + 1 - 9/4$

$\left(\frac{3}{2}\right)^2$

$\boxed{y = (x + 3/2)^2 - 5/4}$

When completing the square, we **must** factor out leading coefficients $\neq 1$ from the x terms. Be careful with balancing the completion of the square!

ex. convert each quadratic to vertex form:

$y = \underbrace{3x^2 - 12x} + 6$

$y = 3(x^2 - 4x + \underline{4}) + 6 - 12$

$\left(\frac{4}{2}\right)^2$

$\boxed{y = 3(x-2)^2 - 6}$

$f(x) = -x^2 + 10x - 20$

$f(x) = -(x^2 - 10x + 25) - 20 + 25$

$\left(\frac{10}{2}\right)^2$

$\boxed{f(x) = -(x-5)^2 + 5}$

$f(x) = -4x^2 - 20x + 1$

$-4\left(-25/4\right)$

$f(x) = -4\left(x^2 + 5x + \frac{25}{4}\right) + 1 + 25$

$\left(\frac{5}{2}\right)^2$

$\boxed{f(x) = -4\left(x + 5/2\right)^2 + 26}$

$y = 2x^2 + 7x$

$2\left(-\frac{49}{16}\right)$

$y = 2\left(x^2 + 7/2\,x + \frac{49}{16}\right) - 49/8$

$\left(\frac{7/2}{2}\right)^2$

$\boxed{y = 2\left(x + 7/4\right)^2 - 49/8}$

ex. The leadership class is planning a fundraising with a professional photographer taking portraits of individuals or groups. The class gets to charge and keep a session fee for each individual or group photo sessions. Last year, they charged a $10 session fee and 400 sessions were booked. In considering what price they should charge this year, the class estimates that for every $1 increase in the price, they expect to have 20 fewer sessions booked.

a) Write a function to model this situation.

Let R = revenue

C = # of $\$1$ price increases

Revenue = cost × bookings

$R(c) = (10 + c)(400 - 20c)$

$R(c) = 10(400 - 20c) + c(400 - 20c)$

$= 4000 - 200c + 400c - 20c^2$

$\boxed{R(c) = -20c^2 + 200c + 4000}$

b) What is the maximum revenue they can expect based on these estimates? What session fee will give that maximum?

* need vertex

$R(c) = -20(c^2 - 10c + 25) + 4000 + 500$

$R(c) = -20(c - 5)^2 + 4500$

vertex $(5, 4500)$

Max Revenue $4500 with a session fee of $15

c) How can you verify the solution?

graph the standard form with tech & find vertex using max finder

Practice: pg. 192/# 2 – 4abc, 6abc, 7cde, 8 – 10, 12 – 14, 18, 19, 21, 23