

Unit 2: Quadratic Functions

2.3 Quadratic Functions in Standard Form

Vertex form quadratic: $f(x) = a(x - p)^2 + q$ where $a, p, q \in \mathbb{R}; a \neq 0$

Standard form quadratic: $f(x) = ax^2 + bx + c$ where $a, b, c \in \mathbb{R}; a \neq 0$

- a determines the shape of the parabola & the direction of opening
& influences the horizontal & vertical translations
- b influences the horizontal & vertical translations
- c represents the y-intercept of the parabola
& influences the vertical translation

* note: $p = \frac{-b}{2a}$

ex. graph the parabola $f(x) = 2x^2 - 3x - 5$ on a calculator and state:

a) vertex $(0.75, -6.125)$

b) axis of Symmetry $x = 0.75$

c) direction of opening up

d) y-intercept $y = -5$

e) x-intercepts

$$x = -1 \leftarrow x = 2.5$$

f) domain $\{x \mid x \in \mathbb{R}\}$

g) range $\{y \mid y \geq -6.125, y \in \mathbb{R}\}$

ex. graph the parabola $f(x) = -\frac{1}{2}x^2 - 5x + 10$ on a calculator and state:

a) vertex $(-5, 22.5)$

b) axis of Symmetry $x = -5$

c) direction of opening down

d) y-intercept $y = 10$

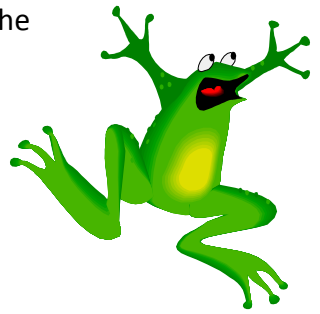
e) x-intercepts

$$x = -11.71 \leftarrow x = 1.71$$

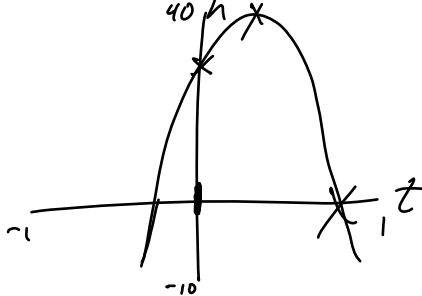
f) domain $\{x \mid x \in \mathbb{R}\}$

g) range $\{y \mid y \leq 22.5, y \in \mathbb{R}\}$

ex. A frog sitting on a rock jumps into a pond. The height, h , in centimeters of the frog above the surface of the water as a function of time, t , in seconds, since it jumped can be modeled by the function $h(t) = -490t^2 + 150t + 25$. Where appropriate, answer the following questions to the nearest tenth.



- a) Graph the function, sketch.



- b) What is the y-intercept? What does it represent in this situation?

$y = 25$ at $t = 0s$ the frog is 25 cm above the pond

- c) What maximum height does the frog reach? When does it reach that height?

vertex $(0.2, 36.5)$ max height 36.5 cm
at time 0.2 s

- d) When does the frog hit the surface of the water?

$h(t) = 0$
ie. x-intercept $t = 0.4s$

- e) What are the domain and range in this situation?

$D: \{t \mid 0 \leq t \leq 0.4, t \in \mathbb{R}\}$
 $R: \{h \mid 0 \leq h \leq 36.5, h \in \mathbb{R}\}$

- f) How high is the frog 0.25 seconds after it jumps?

$h(0.25) = 31.9 \text{ cm}$

ex. A rancher has 100 m of fencing available to build a rectangular corral. Answer the following questions without the aid of the graphing features of a graphing calculator.

- a) Write a quadratic function in standard form to represent the area of the corral.



$$A = l \times w$$

$$A(w) = (50 - w)(w)$$

$$A(w) = 50w - w^2$$

$$A(w) = -w^2 + 50w$$

$$2l + 2w = 100$$

$$2l = 100 - 2w \therefore l = 50 - w$$

- b) What are the coordinates of the vertex? What does the vertex represent in this situation?

$$p = \frac{-b}{2a}$$

$$= \frac{-50}{2(-1)}$$

$$= 25$$

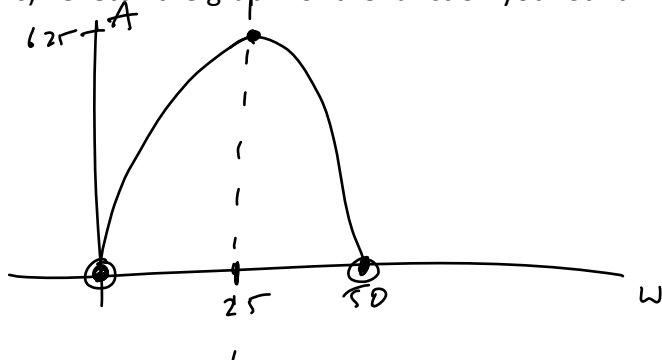
$$q = A(25)$$

$$= -(25)^2 + 50(25)$$

$$= 625$$

vertex $(25, 625)$
max area of 625 m^2
when the width is 25 m

- c) Sketch the graph for the function you found in a).



- d) Determine the domain and range.

$$D: \{w \mid 0 < w < 50, w \in \mathbb{R}\}$$

$$R: \{A \mid 0 < A \leq 625, A \in \mathbb{R}\}$$

- e) Identify any assumptions you made in modeling this situation mathematically.

- using all the fencing
- all dimensions are possible