PMA12
Mr. C. Sdoutz

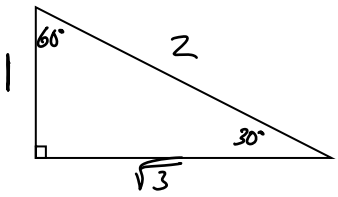
Date:_____

Unit 6: Standard Position Trigonometry

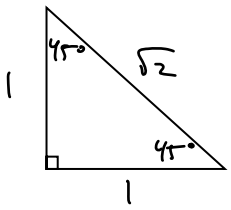
6.3 Special Triangles, Exact Values, & Quadrantal Angles

Special Triangles:

$30° - 60° - 90°$



$45° - 45° - 90°$



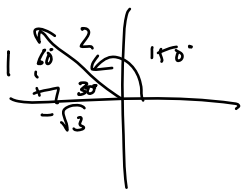
These triangles allow us to produce exact values for all the trig ratios (without use of a calculator) involving $30°, 45° \& 60°$:

	$30°$	$45°$	$60°$
$\sin\theta = \frac{o}{h}$	$\sin 30° = \frac{1}{2}$	$\sin 45° = \frac{1}{\sqrt{2}}$	$\sin 60° = \frac{\sqrt{3}}{2}$
$\cos\theta = \frac{a}{h}$	$\cos 30° = \frac{\sqrt{3}}{2}$	$\cos 45° = \frac{1}{\sqrt{2}}$	$\cos 60° = \frac{1}{2}$
$\tan\theta = \frac{o}{a}$	$\tan 30° = \frac{1}{\sqrt{3}}$	$\tan 45° = 1$	$\tan 60° = \sqrt{3}$

Can also be used to produce exact values for all the trig ratios (without use of a calculator) for standard position angles whose reference angles are $30°, 45°$ or $60°$:

ex. Find the exact value of:

$\sin 150°$



S | A
T | C

QII $\sin\theta = +$

$\sin 150° = \sin 30°$

$= \frac{1}{2}$

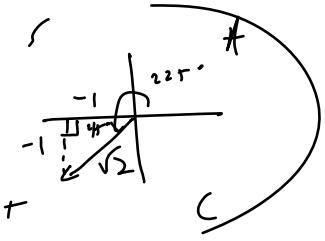
$\cos 150°$

$\cos\theta = -$

$\cos 150° = -\cos 30°$

$= -\frac{\sqrt{3}}{2}$

$\sin 225°$



QIII $\sin\theta = -$

$\sin 225° = -\sin 45°$

$= -1/\sqrt{2}$

$\tan 225°$

QIII $\tan\theta = +$

$\tan 225° = \tan 45°$

$= 1$

ex. A tree casts a shadow 8 m long when the angle of elevation of the sun is 60°. Calculate the exact height of the tree.

$\tan 60° = \frac{x}{8}$

$x = 8 \tan 60°$

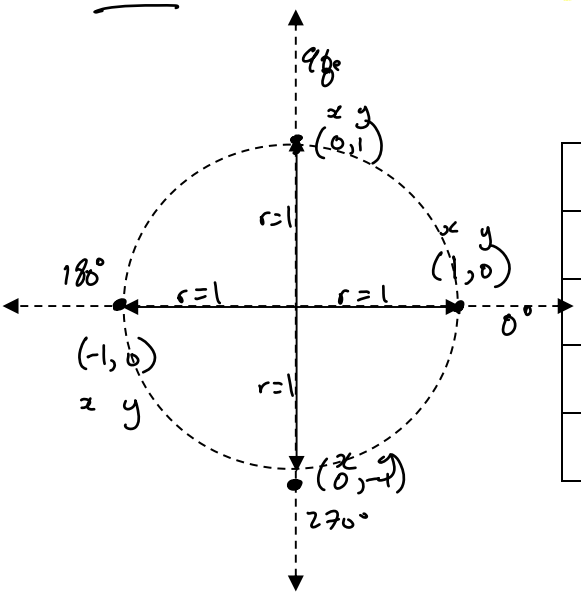
$= 8(\sqrt{3})$

$= 8\sqrt{3}$ m

Quadrantal Angles: An angle in standard position whose terminal arm lies on one of the axis

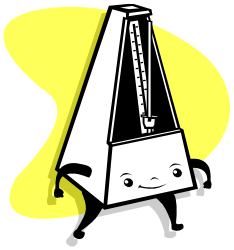
$0°, 90°, 180°, 270°$

We can also determine exact trigonometric ratio values for quadrantal angles by using a *unit circle* (terminal arm length constant $r = 1$).

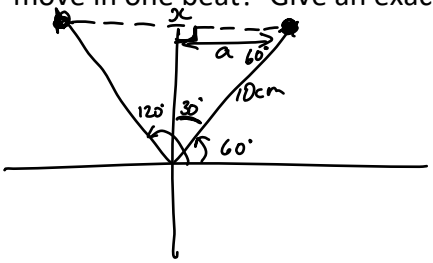


	$\sin\theta = \frac{y}{r}$	$\cos\theta = \frac{x}{r}$	$\tan\theta = \frac{y}{x}$
$0°$	$\sin 0° = \frac{0}{1} = 0$	$\cos 0° = \frac{1}{1} = 1$	$\tan 0° = \frac{0}{1} = 0$
$90°$	$\sin 90° = \frac{1}{1} = 1$	$\cos 90° = \frac{0}{1} = 0$	$\tan 90° = \frac{1}{0} = \infty$
$180°$	$\sin 180° = \frac{0}{1} = 0$	$\cos 180° = \frac{-1}{1} = -1$	$\tan 180° = \frac{0}{-1} = 0$
$270°$	$\sin 270° = \frac{-1}{1} = -1$	$\cos 270° = \frac{0}{1} = 0$	$\tan 270° = \frac{-1}{0} = \infty$

Note: ∞ = undefined



ex. Allie is learning to play the piano. Her teacher uses a metronome to help her keep time. The pendulum arm of the metronome is 10 cm long. For one particular tempo, the setting results in the arm moving back and forth from a start position of 60° to 120°. What horizontal distance does the tip of the arm move in one beat? Give an exact value.



$x = 2a$

$\sin 30° = \frac{a}{10}$

$a = 10 \sin 30°$

$= 10\left(\frac{1}{2}\right)$

$= 5$

$x = 2(5) =$ **10 cm**

Practice: pg. 83/ # 8, 11, 13

pg. 96/ # 2, 9, 10, 18, 19, 20