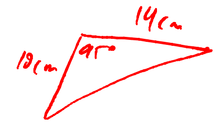
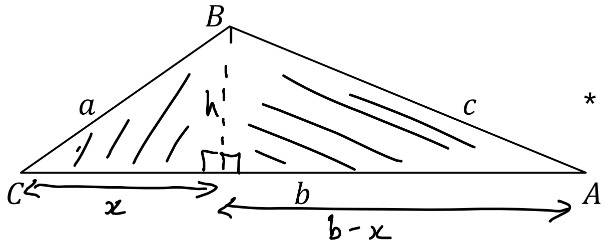
Unit 2A: Non-Right Triangle Trigonometry

2A.4 The Cosine Law (text 2.4)



Recall from 2A.2 (The Sine Law), in an Side-Angle-Side (SAS) situation the Sine Law cannot be used.

Consider ΔABC, with given measures for a, $\angle A$, & c. How can we calculate the length of the final side?



* drop an altitude from B to create two right triangles

Because these are right triangles, we can use the Pythagorean Theorem:

$$h^2 + x^2 = a^2 \qquad -x^2 \quad -x^2$$
$$h^2 = a^2 - x^2$$

$$h^2 + (b-x)^2 = c^2 \qquad -(b-x)^2 \quad -(b-x)^2$$
$$h^2 = c^2 - (b-x)^2$$

Expanding out the binomial term,

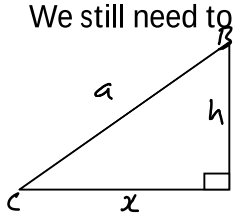
$$\therefore a^2 - x^2 = c^2 - (b-x)^2$$
$$a^2 - x^2 = c^2 - (b^2 - 2bx + x^2)$$
$$a^2 - \cancel{x^2} = c^2 - b^2 + 2bx - \cancel{x^2}$$

$$+(b-x)^2$$
$$(b-x)(b-x)$$
$$b(b-x) - x(b-x)$$
$$b^2 - bx - bx + x^2$$
$$b^2 - 2bx + x^2$$

Isolating the c^2 term,

$$c^2 = a^2 + b^2 - 2bx$$

We still need to replace the x term. If we consider the left side right triangle,



$$\cos C = \frac{x}{a}$$
$$x = a\cos C$$

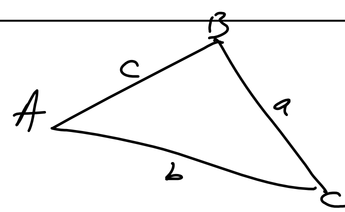
and the formula becomes:

$$c^2 = a^2 + b^2 - 2b(a\cos C)$$
$$c^2 = a^2 + b^2 - 2ab\cos C$$

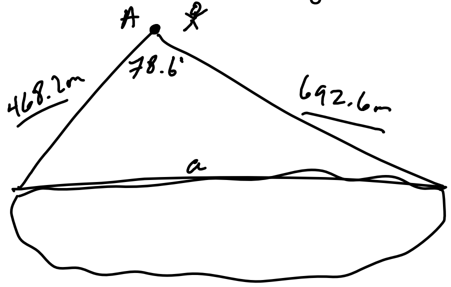
The Cosine Law In any triangle ABC

$$\boxed{c^2 = a^2 + b^2 - 2ab\cos C}$$ * on sheet

$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$a^2 = b^2 + c^2 - 2bc\cos A$$



ex. A surveyor needs to find the length of a swampy area near Fishing Lake, Manitoba. The surveyor sets up her transit at a point A. She measures the distance to one end of the swamp as 468.2 m, the distance to the opposite end of the swamp as 692.6 m, and the angle of sight between the two as 78.6°. Determine the length of the swampy area, to the nearest tenth of a metre.



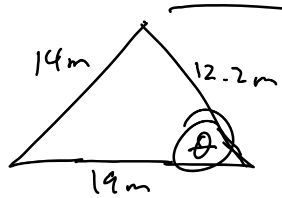
$a^2 = 468.2^2 + 692.6^2 - 2(468.2)(692.6)\cos 78.6°$

$a^2 = 570\,715.2054....$

$a = \sqrt{ans}$

$= \boxed{755.5\text{ m}}$

ex. The Lions' Gate Bridge has been a Vancouver landmark since it opened in 1938. It is the longest suspension bridge in Western Canada. The bridge is strengthened by triangular braces. Suppose one brace has side lengths 14 m, 19 m, and 12.2 m. Determine the measure of the angles opposite the 14-m side to the nearest degree.



$14^2 = 12.2^2 + 19^2 - 2(12.2)(19)\cos\theta$

$196 = 509.84 - 463.6\cos\theta$

$-509.84 \qquad -509.84$

$\frac{-313.84}{-463.6} = \frac{-463.6\cos\theta}{-463.6}$

$0.67696.... = \cos\theta$

$\theta = \cos^{-1}(ans)$

$\theta = 47°$

$c^2 = a^2 + b^2 - 2ab\cos C$

$c^2 - a^2 - b^2 = -2ab\cos C$

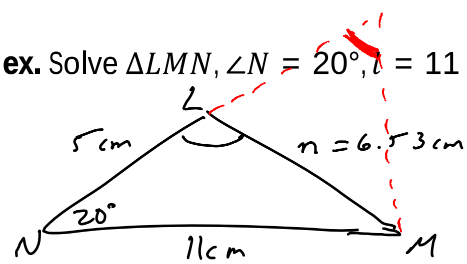
$\frac{c^2 - a^2 - b^2}{-2ab} = \cos C$

$C = \cos^{-1}\left(\frac{c^2 - a^2 - b^2}{-2ab}\right)$

$\theta = \cos^{-1}\left(\frac{(14^2 - 12.2^2 - 19^2)}{(-2(12.2)(19))}\right)$

$\theta = 47°$

ex. Solve ΔLMN, $\angle N = 20°$, $l = 11\ cm$, and $m = 5\ cm$. Side lengths and angles to the nearest tenth.



$n^2 = 5^2 + 11^2 - 2(5)(11)\cos 20°$

$n = \sqrt{ans}$

$n = \underline{6.53}$

Find L

~~$\frac{\sin L}{11} = \frac{\sin 20}{6.53}$~~

~~$\sin L = \frac{11\sin 20}{6.53}$~~

~~$L = \sin^{-1}(ans)$~~

~~$L = 35.18°$~~

ambiguous case

$M^2 = 5^2 + 6.53^2 - 2(5)(6.53)\cos L$

$L = 144.79°$ ✓

$180 - 35.18$

$M = 180° - 20 - 144.79$

$$\boxed{\begin{array}{l} n = 6.5\text{ cm} \\ M = 15.2° \\ L = 144.8° \end{array}}$$

Practice: pg. 119/ # 1 – 3ab, 4ae, 5, 7 – 10