PreCalculus 11 Date _____

Unit 5: Non-Right Triangle Trigonometry

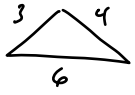
5.3 The Sine Law – Ambiguous Case (text 2.3)

Information to construct a triangle:

AAA
Angle Angle Angle

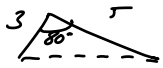
infinite

SSS
side side side



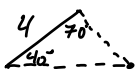
1 triangle

SAS
side Angle side



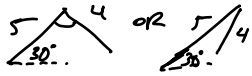
1 triangle

ASA
Angle side Angle



1 triangle

SSA
side side Angle



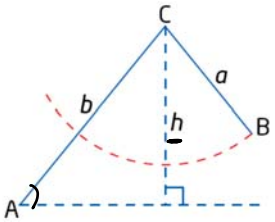
???

Ambiguous Case: In a SSA situation the triangle is unclear.
There maybe – no such triangle
– one triangle
– two triangles

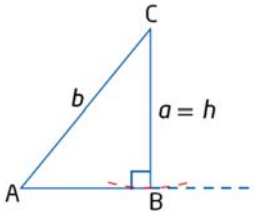
Applet (click)

Suppose you are given the measure of $\angle A, a, \& b$ of ΔABC. $\sin A = \frac{h}{b}$ $\therefore h = b\sin A$

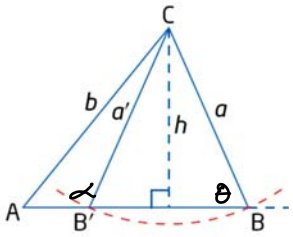
Case 1: $\angle A$ acute – 4 possibilities



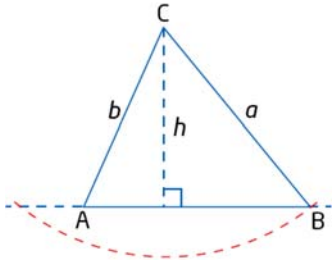
$a < h$
$a < b\sin A$
No solution



$a = h$
$a = b\sin A$
1 right Δ solution

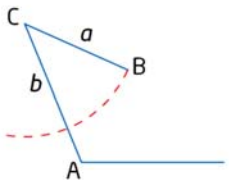


$h < a < b$
$b\sin A < a < b$
2 solutions
$\theta + \alpha = 180°$

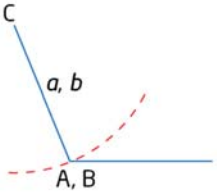


$a \geq b$
1 solution

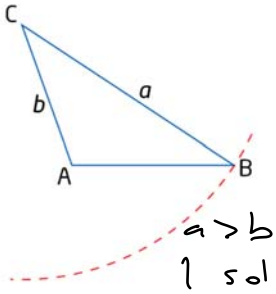
Case 2: $\angle A$ obtuse – 3 possibilities



$a < b$
no solution

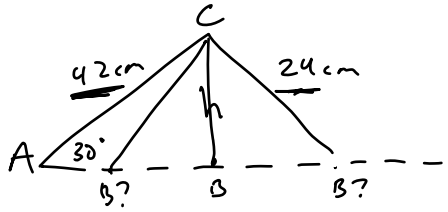


$a = b$
no solution



$a > b$
1 solution

ex. Solve $\Delta ABC, \angle A = 30°, a = 24\ cm$, and $b = 42\ cm$. Angles to the nearest degree and side lengths to the nearest tenth.

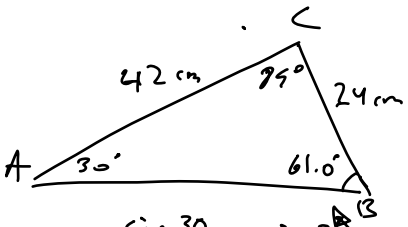


$h = 42 \sin 30$

$h = 21 cm$

$21 < 24 < 42$

$\therefore$ 2 triangles



$\frac{\sin 30}{24} = \frac{\sin B}{42}$

$\sin B = \frac{42 \sin(30)}{24}$

$B = \sin^{-1}(as)$

$B = 61.0°$

$C = 180 - 30 - 61$

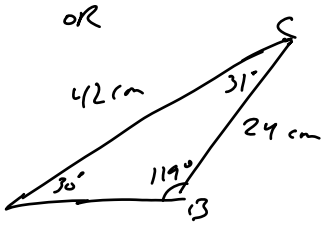
$C = 89°$

$\frac{\sin 89}{c} = \frac{\sin 30}{24}$

$c = \frac{24 \sin 89}{\sin 30}$

$c = 48.0\ cm$

OR



$B = 180 - 61.0 = 119°$

$C = 180 - 30 - 119$

$C = 31°$

$\frac{\sin 31}{c} = \frac{\sin 30}{24}$

$c = \frac{24 \sin 31}{\sin 30}$

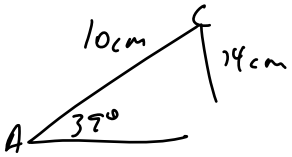
$c = 24.7 cm$

$B = 61°$ $C = 89°$ $c = 48.0 cm$

OR

$B = 119°$ $C = 31°$ $c = 24.7 cm$

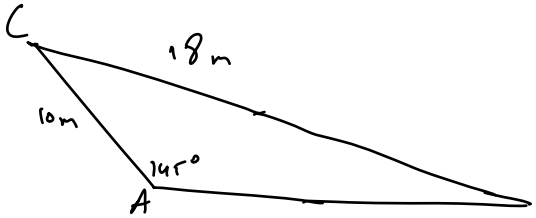
ex. Determine whether there are 1, 2 or no solutions for $\Delta ABC, \angle A = 39°, a = 14\ cm$, and $b = 10\ cm$



$14 > 10$

$\therefore$ 1 solution

ex. Determine whether there are 1, 2 or no solutions for $\Delta ABC, \angle A = 145°, a = 18\ m$, and $b = 10\ m$



Obtuse

$18 > 10$

$\therefore$ 1 solution

Practice: pg. 108/ # 6, 8, 9, 11, 17, 22