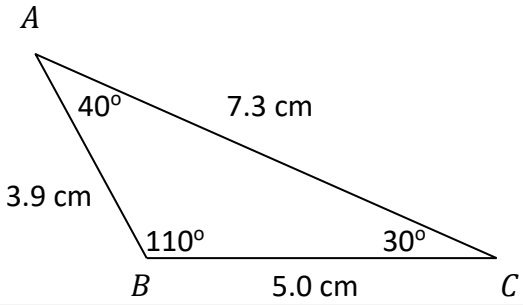
PreCalculus 11 Date _____

Unit 5: Non-Right Triangle Trigonometry

5.2 The Sine Law

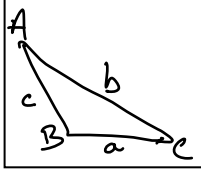
Consider ΔABC.

For each angle calculate the ratio of the sine of the angle to the length of the side opposite.



$\frac{\sin 30^\circ}{3.9} = 0.13$ $\frac{\sin 40^\circ}{5.0} = 0.13$ $\frac{\sin 110^\circ}{7.3} = 0.13$

The **sine law** is a relationship between the sides and angles in any triangle. Let ΔABC be any triangle, where a, b, and c represent the measures of the sides opposite $\angle A$, $\angle B$, and $\angle C$, respectively.



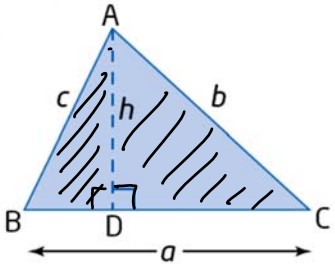
$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

* on formula sheet

or $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Proof:

In ΔABC, draw an altitude $AD \perp BC$. Let $AD = h$.



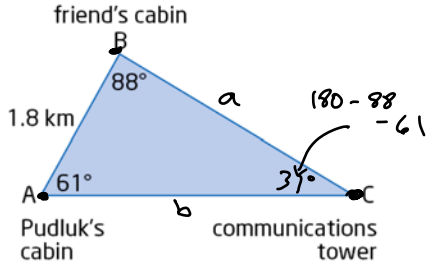
ΔABD $\sin B = \frac{h}{c}$ ΔADC $\sin C = \frac{h}{b}$

$h = c\sin B$ $h = b\sin C$

$\therefore c\sin B = b\sin C$

$\frac{\sin B}{b} = \frac{\sin C}{c}$

ex. Pudluk's family and his friend own cabins on the Kalit River in Nunavut. Pudluk and his friend wish to determine the distance from Pudluk's cabin to the store on the edge of town. They know that the distance between their cabins is 1.8 km. Using a transit, they estimate the measures of the angles between their cabins and the communications tower near the store, as shown in the diagram.



a) Determine the distance from Pudluk's cabin to the store, to the nearest tenth of a kilometre.

b) Determine the distance from Pudluk's friend's cabin to the store.

a) find b

$\frac{\sin 88^\circ}{b} = \frac{\sin 31^\circ}{1.8}$ ← must have one angle/side pair to use sine law

$b = \frac{1.8\sin 88^\circ}{\sin 31^\circ}$

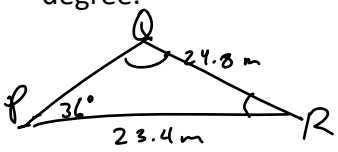
$= 3.5$ km

b) $\frac{\sin 61^\circ}{a} = \frac{\sin 31^\circ}{1.8}$

$a = \frac{1.8\sin 61}{\sin 31}$

$= 3.1$ km

ex. In ΔPQR, $\angle P = 36°, p = 24.8\ m$, and $q = 23.4\ m$. Determine the measure of $\angle R$, to the nearest degree.



find $\angle Q$ first

$$\frac{\sin Q}{23.4} = \frac{\sin 36°}{24.8}$$

$$\sin Q = \frac{23.4 \sin 36°}{24.8} \rightarrow Q = \sin^{-1}\left(\frac{23.4 \sin 36}{24.8}\right)$$

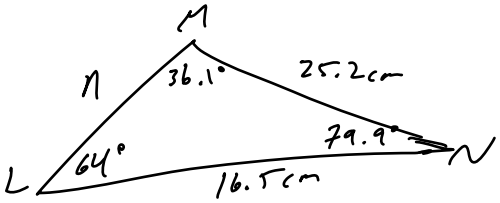
$Q = \sin^{-1}(\text{ans})$

$= 33.7°$

$\therefore \angle R = 180° - 36° - 33.7°$

$\angle R = 110°$

ex. Solve $\Delta LMN, \angle L = 64°, l = 25.2\ cm$, and $m = 16.5\ cm$. Side lengths to the tenth and angle measures to the nearest degree.



$$\frac{\sin M}{16.5} = \frac{\sin 64°}{25.2}$$

$$\sin M = \frac{16.5 \sin 64°}{25.2}$$

$M = \sin^{-1}(\text{ans})$

$M = 36.1°$

$\angle M = 36°$

$\angle N = 180 - 64 - 36.1$

$N = 79.9°$

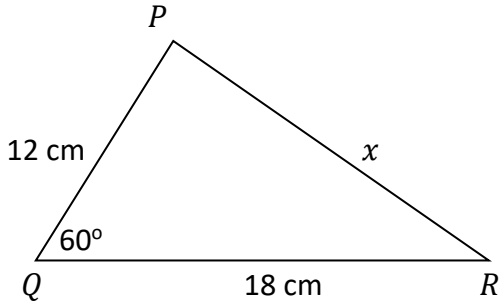
$\angle N = 80°$

$$\frac{\sin 79.9}{n} = \frac{\sin 64}{25.2}$$

$$n = \frac{25.2 \sin 79.9}{\sin 64}$$

$n = 27.6\ cm$

ex. In the triangle shown, can the sine law be used to determine the value of x? $\angle P$? $\angle R$? Explain.



$$\frac{\sin 60°}{x} = \frac{\sin P}{18} = \frac{\sin R}{12}$$

No, any sine law ratio would have too many variables because there is no known angle/side pair.

Practice: pg. 108/ # 1ac, 2, 3, 4ab, 5ab, 10, 12, 13, 15, 24abc