PreCalculus 11 Date ________________

Unit 1: Sequences & Series

1.5 Infinite Geometric Series

Consider the series 4 + 8 + 16 + 32 + ⋯ $t_1 = 4$ $r = 2$

$S_n = \frac{t_1(r^n - 1)}{r-1}$

$S_n = \frac{4(2^n - 1)}{\cancel{2-1}}$ *use graphing calculator table property to fill in the table

n	3	5	7	9	11	13	15	17
S_n	28	124	508	2044	8188	32764	131068	524254

The partial sums continue to increase

Divergent Series: A series with an infinite number of terms in which the sequence of partial sums does not approach any fixed value.
ie. it has no finite sum

Consider the series $4 + 2 + 1 + \frac{1}{2} + \frac{1}{4} + \cdots$ $t_1 = 4$, $r = 1/2$

$S_n = \frac{4((1/2)^n - 1)}{1/2 - 1} = -8((1/2)^n - 1)$ *use graphing calculator table property to fill in the table

n	3	5	7	9	11	13	15	17
S_n	7	7.75	7.9375	7.9844	7.9961	7.999	7.9998	7.9999

The partial sums approach a value of 8

Convergent Series: A series with an infinite number of terms in which the sequence of partial sums approaches a finite value

*only when $-1 < r < 1$

Consider the formula $S_n = \frac{t_1(r^n-1)}{r-1}$. What happens to the value of r^n as *n* gets larger and larger in a convergent series(ie. approaches infinity)?

$$S_\infty = \frac{t_1(r^\infty - 1)}{r-1} = \frac{t_1(0-1)}{r-1} = \frac{-t_1}{r-1} = \frac{t_1}{1-r}$$

Formula for the Sum of an Infinite Geometric Series

$$S_\infty = \frac{t_1}{1-r}$$

S_∞ = infinite sum of a convergent series
t_1 = 1st term
r = common ratio $-1 < r < 1$

ex. Find the sum of the series: $8 + 4 + 2 + 1 + \frac{1}{2} + \cdots$ $r = \frac{1}{2}$ ∴ convergent

$$S_\infty = \frac{t_1}{1-r} = \frac{8}{1-\frac{1}{2}} = \boxed{16}$$

How can a list of infinite numbers add to a definite and finite value? Think of the situation in reverse. We could start with the number 16 and split it into a sum:

16 = 8 + 8

Then we can split the last number in that sum again:

16 = 8 + 4 + 4

And repeat:

16 = 8 + 4 + 2 + 2

16 = 8 + 4 + 2 + 1 + 1

16 = 8 + 4 + 2 + 1 + ½ + ½

If we continue this infinitely we get the infinite geometric series we originally started with in the previous example:

$8 + 4 + 2 + 1 + \frac{1}{2} + \cdots$

ex. Determine whether each infinite geometric series converges or diverges. Calculate the sum, if it exists.

a) $1 + \frac{1}{5} + \frac{1}{25} + \cdots$

$r = \frac{1}{5}$

converges

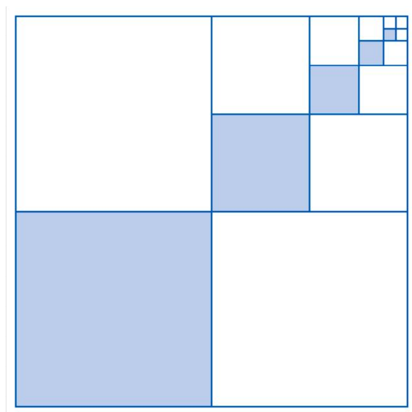
$$S_\infty = \frac{1}{1-\frac{1}{5}} = \frac{1}{\frac{4}{5}} = \boxed{\frac{5}{4}}$$

b) 8 – 12 + 18 ...

$r = -\frac{3}{2}$

divergent ∴ no finite sum

ex. Assume that each shaded square represents $\frac{1}{4}$ of the area of the larger square bordering two of its adjacent sides and that the shading continues indefinitely in the indicated manner.



a) Write the series of terms that would represent this situation.

$\frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \cdots$

b) How much of the total area of the largest square is shaded?

$r = \frac{1}{4}$ convergent

$$S_\infty = \frac{\frac{1}{4}}{1-\frac{1}{4}} = \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{\cancel{4}} \times \frac{\cancel{4}}{3} = \boxed{\frac{1}{3}}$$

Practice: pg. 63/ # 1ace, 2ace, 5ab, 6 – 10, 13, 15, 17