

Unit 1: Sequences & Series

1.4 Geometric Series

Geometric Series: sum of the terms of a geometric sequence

ex 2, 10, 50, 250
sequence

2 + 10 + 50 + 250
series

Formulae for the Sum of a Geometric Series

$$S_n = \frac{t_1 (r^n - 1)}{r - 1} \quad \text{or} \quad S_n = \frac{r t_n - t_1}{r - 1} \quad r \neq 1$$

S_n = sum of first n terms

t_1 = 1st term

r = common ratio

n = # of terms

t_n = n^{th} term (last term added)

ex. Determine the sum of the first 10 terms of each geometric series.

a) 4 + 12 + 36 + ...

$t_1 = 4$ $r = 3$ $n = 10$

$$S_{10} = \frac{4(3^{10} - 1)}{3 - 1}$$

$$S_{10} = 118096$$

b) $t_1 = 5, r = \frac{1}{2}, n = 10$

$$S_{10} = \frac{5((\frac{1}{2})^{10} - 1)}{\frac{1}{2} - 1}$$

$$= -10((\frac{1}{2})^{10} - 1)$$

$$S_{10} = \frac{5115}{512}$$

ex. Determine the sum of each geometric series.

a) $\frac{1}{27} + \frac{1}{9} + \frac{1}{3} + \dots + 729$

$$S_n = \frac{r t_n - t_1}{r - 1}$$

$t_1 = \frac{1}{27}$

$$r = \frac{\frac{1}{3}}{\frac{1}{27}} = \frac{1}{3} \times \frac{27}{1} = 9$$

$$S_n = \frac{9(729) - \frac{1}{27}}{9 - 1}$$

$$= \frac{29524}{8}$$

b) 4 - 16 + 64 - ... - 65536

$t_1 = 4$

$r = -4$

$$S_n = \frac{-4(-65536) - 4}{-4 - 1}$$

$$= -52428$$

ex. The Western Scrabble™ Network is an organization whose goal is to promote the game of Scrabble™. It offers Internet tournaments throughout the year that WSN members participate in. The format of these tournaments is such that each game has two players and the losers of each round are eliminated from the next round. The winners continue to play until a final match determines the champion. If there are 256 entries in an Internet Scrabble™ tournament, what is the total number of matches that will be played in the tournament?

$$\begin{array}{l} \text{1st round} \\ \frac{256}{2} = 128 \text{ games} \end{array}$$

$$\begin{array}{l} \text{2nd round} \\ 64 \text{ games} \end{array} \quad \dots$$

$$\begin{array}{l} \text{Final round} \\ \underline{\underline{1 \text{ game}}} \end{array}$$

$$t_1 = 128$$

$$r = \frac{1}{2}$$

$$t_n = 1$$

$$S_n = \frac{\frac{1}{2}(1) - 128}{\cancel{\frac{1}{2}} - \frac{1}{2}}$$

$$= -2 \left(\frac{1}{2} - 128 \right)$$

$$= -1 + 256$$

$$= \boxed{255 \text{ games}}$$

Practice: p.53/ # 2 – 4 ac, 5, 6, 9, 10, 13, 14, 19