

Unit 1: Sequences & Series

1.3 Geometric Sequences

Geometric Sequence: a sequence in which the ratio of consecutive terms is constant

Common Ratio: ie. multiply by a constant value to generate the terms
The ratio of consecutive terms in a geometric sequence

ex. 1, 2, 4, 8, ...

$$r = \frac{2}{1} = \frac{4}{2} = \frac{8}{4}$$

$$r = 2$$

5, -10, 20, -40, ...

$$r = -2$$

8, 4, 2, 1, ...

$$r = \frac{1}{2}$$

$t_1, t_1 r, t_1 r^2$

General Term of a Geometric Sequence

$$t_n = t_1 r^{n-1}$$

t_1 = 1st term
 r = common ratio
 n = # of terms

ex. Single-celled organisms, such as bacteria, reproduce by splitting in two so that one cell gives rise to 2, then 4, then 8 cells and so on. Suppose there are 10 bacteria currently present in a sample.

Determine the general term that relates the number of bacteria to the doubling period of the bacteria. State the values for t_1 and r in the geometric sequence.

10, 20, 40, ...

$$\boxed{t_1 = 10}$$

$$r = 2$$

$$t_n = t_1 r^{n-1}$$

$$\boxed{t_n = 10(2)^{n-1}}$$

ex. Sometimes photocopiers are used for enlargements and reductions. Suppose the actual length of a photograph is 25 cm and the smallest size that a copier can make is 67% of the original. What is the shortest possible length of the photograph after 5 reductions? Express your answer to the nearest tenth of a centimeter.

25 cm, 16.75 cm, ...
x. 67

$$t_1 = 25$$

$$r = 0.67$$

$$t_6 = ?$$

$$t_6 = 25(0.67)^5$$

$$= \boxed{3.4 \text{ cm}}$$

ex. In a geometric sequence, the third term is 54 and the sixth term is -1458. Determine the values of t_1 and r and list the first three terms in the sequence.

$$\begin{aligned}
 t_n &= t_1 r^{n-1} & t_3 &= 54 & t_6 &= -1458 \\
 54 &= t_1 r^{2} & -1458 &= t_1 r^{5}
 \end{aligned}$$

$$\begin{array}{r}
 54 = t_1 (-3)^2 \\
 54 = 9t_1 \\
 \boxed{t_1 = 6}
 \end{array}$$

$$\begin{array}{r}
 -1458 = t_1 r^5 \\
 54 = t_1 r^2 \\
 \hline
 -27 = r^3 \\
 \sqrt[3]{-27} = r \\
 \boxed{r = -3}
 \end{array}$$

← elimination by division

$$\boxed{6, -18, 54}$$

ex. Determine the number of terms in the geometric sequence: 2, 6, 18, ..., 13122

$$\begin{aligned}
 t_n &= t_1 r^{n-1} & t_1 &= 2 & t_n &= 13122 \\
 & & r &= 3 & n &= ?
 \end{aligned}$$

$$\begin{aligned}
 \frac{13122}{2} &= \frac{2(3)^{n-1}}{2} \\
 6561 &= 3^{n-1} \\
 3^8 &= 3^{n-1} \\
 \therefore 8 &= n-1 \\
 n &= 9
 \end{aligned}$$

$$\boxed{9 \text{ terms}}$$

Practice: p.39 / # 1abc, 3bd, 5cd, 6abc, 7, 8, 12, 17, 18, 23, 25