PreCalculus 11 Date ________________

Unit 1: Sequences & Series
1.2 Arithmetic Series

Arithmetic Series: sum of terms of an arithmetic sequence

ex. 2, 4, 6, 8 sequence

2 + 4 + 6 + 8 series

How would we find the sum of all the terms in an arithmetic series?
ex. Gauss Method

Determine the sum of the series $\boxed{1} + 2 + 3 + \cdots + 98 + 99 + 100$ ← 100 terms

ie. $n = 100$

$$\begin{array}{r} 1 + 2 + 3 + \cdots + 98 + 99 + 100 \\ + \quad \boxed{100} + 99 + 98 + \cdots + 3 + 2 + 1 \\ \hline \underbrace{101 + 101 + 101 + \cdots + 101 + 101 + 101} \end{array}$$

$101(100)$

$= 10{,}100$ ← double the sum

$\div 2$

$= 5050$

$t_n = t_1 + (n-1)d$

$$\frac{n(t_1 + t_n)}{2} \quad \text{OR} \quad \frac{n}{2}(t_1 + t_1 + (n-1)d)$$

$$\frac{n}{2}(t_1 + \boxed{t_n}) \qquad \frac{n}{2}(2t_1 + (n-1)d)$$

Formulae for Sum of an Arithmetic Series

$$S_n = \frac{n}{2}(t_1 + t_n) \quad \text{OR} \quad S_n = \frac{n}{2}(2t_1 + (n-1)d)$$

where S_n = sum of first n terms

n = number of terms

d = common difference

t_1 = 1st term

t_n = n^{th} term (last term added on) ex. Male fireflies flash in various patterns to signal location or to ward off predators. Different species of fireflies have different flash characteristics, such as the intensity of the flash, the rate of the flash, and the shape of the flash. Suppose that under certain circumstances, a particular firefly flashes twice in the first minute, four times in the second minute, and six times in the third minute.

a) How many flashes in the 30th minute, assuming the pattern continues?

$t_1 = 2$
$d = 2$

$$t_{30} = 2 + (\cancel{30}^{29} - 1)2$$
$$= \boxed{60 \text{ flashes}}$$

b) What is the total number of flashes in 30 minutes?

$$S_n = \frac{n}{2}(2t_1 + (n-1)d)$$
$$S_{30} = \frac{30}{2}(2(2) + (\cancel{30}^{29}-1)2)$$
$$= 15(4 + 58)$$
$$= 15(62)$$
$$= \boxed{930 \text{ flashes}}$$

OR

$$S_n = \frac{n}{2}(t_1 + t_n)$$
$$S_{30} = \frac{30}{2}(2 + 60)$$
$$= 15(62)$$

$t_{30} = 60$

ex. Determine the sum of the series: 435 + 431 + 427 + ⋯ + 15 + 11

$t_1 = 435$ $\quad n = ?$
$d = -4$
$t_n = 11$

$$t_n = t_1 + (n-1)d$$
$$11 = 435 + (n-1)(-4)$$
$$-424 = (n-1)(-4)$$
$$106 = n - 1$$
$$n = 107$$

$$S_{107} = 107/2\,(435 + 11)$$
$$= 53.5(446)$$
$$= \boxed{23{,}861}$$

ex. The sum of the first two terms of an arithmetic series is 13 and the sum of the first four terms is 46. Determine the sum of the first six terms.

$n = 6$

$t_1 = ?$
$d = ?$

$$S_n = \frac{n}{2}(2t_1 + (n-1)d)$$

$S_2 = 13$

$$13 = \frac{2}{2}(2t_1 + (2-1)d)$$
$$13 = 2t_1 + d$$

$S_4 = 46$

$$46 = \frac{4}{2}(2t_1 + (\cancel{4-1})^{3}d)$$
$$\frac{46}{2} = \cancel{2}(2t_1 + 3d)$$
$$23 = 2t_1 + 3d$$
$$-\ 13 = 2t_1 + d$$
$$10 = 2d$$
$$d = 5$$

$$13 = 2t_1 + 5$$
$$8 = 2t_1$$
$$t_1 = 4$$

$$S_6 = \frac{6}{2}(2(4) + (6-1)5)$$
$$= 3(8 + 25)$$
$$= 3(33)$$
$$\boxed{S_6 = 99}$$

Practice: p. 27/ # 1 – 6 (ac), 7, 9 – 11, 13, 15, 17, 18, 20