

Unit 1: Sequences & Series

1.1 Arithmetic Sequences

A **sequence** is an ordered list of objects. The terms of a sequence are labelled according to their position in the sequence.

- the **first term** is t_1
- the **number of terms** is n
- the **general term** is t_n and is dependent on value of n .

A **finite sequence** has a finite # of terms (sequence stops).
 e.g. $2, 4, 6, 8, 10$ $3, 10, 17, \dots, 73$

An **infinite sequence** has an infinite # of terms (sequence continues forever)
 e.g. $5, 2, -1, -4, \dots$

An **arithmetic sequence** is an ordered list in which the difference between consecutive terms is constant. This constant is called the common difference (d)

The formula for the **general term** helps you find the terms of the sequence

e.g.

numeric

$d = 17$

algebraic

$$t_1 = 5$$

$$t_1 = t_1$$

$$t_2 = 5 + 17 = 22$$

$$t_2 = t_1 + d$$

$$t_3 = 5 + 17 + 17 = 39$$

$$t_3 = t_1 + d + d = t_1 + 2d$$

$$t_4 = 5 + 17 + 17 + 17 = 56$$

$$t_4 = t_1 + 2d + d = t_1 + 3d$$

$$t_n = t_1 + (n-1)d$$

The **general term** of an arithmetic sequence is $t_n = t_1 + (n-1)d$

where t_1 = 1st term

n = number of terms

d = common difference

t_n = n^{th} term OR general term of the sequence

ex. A visual and performing arts group wants to hire a community events leader. The person will be paid \$12 for the first hour of work, \$19 for two hours of work, \$26 for three hours of work, and so on.

a) Find the general term

Hours	1	2	3
Pay	12	19	26

$$t_1 = 12$$

$$d = 7$$

$$t_n = t_1 + (n-1)d$$

$$t_n = 12 + (n-1)7$$

$$= 12 + 7n - 7$$

$$t_n = 7n + 5$$

b) What will the person get paid for 6 h? 10 h?

$$t_6 = 7(6) + 5$$

$$= \$47$$

$$t_{10} = 7(10) + 5$$

$$= \$75$$

ex. In 1955 the Banks Island musk-ox population was 9250. If the population increases by 1650 each year, how long would it take to reach 100 000?

$$t_n = 100000$$

$$d = 1650$$

$$t_1 = 9250$$

$$n = ?$$

$$t_n = t_1 + (n-1)d$$

$$100000 = 9250 + (n-1)1650$$

$$100000 = 9250 + 1650n - 1650$$

$$100000 = 1650n + 7600$$

$$-7600$$

$$92400 = 1650n$$

$$\frac{92400}{1650} = \frac{1650n}{1650}$$

$$n = 56 \text{ years}$$

OR

$$100000 = 9250 + (n-1)1650$$

$$-9250$$

$$90750 = (n-1)1650$$

$$\frac{90750}{1650} = \frac{(n-1)1650}{1650}$$

$$55 = n-1$$

$$n = 56 \text{ years}$$



ex. Find the first three terms of the arithmetic sequence with $t_5=14$ and $t_{11}=-10$.

$$t_n = t_1 + (n-1)d$$

$$\begin{aligned} t_5 = 14 &\Rightarrow 14 = t_1 + \cancel{(5-1)}^4 d \\ t_{11} = -10 &\Rightarrow -10 = t_1 + \cancel{(11-1)}^{10} d \end{aligned}$$

$$\begin{array}{r} 24 = 6d \\ \underline{-6} \quad \underline{-6} \end{array}$$

$$d = -4$$

$$14 = t_1 + 4(-4)$$

$$14 = t_1 - 16$$

$$t_1 = 30$$

$$\boxed{30, 26, 22} \quad \begin{array}{r} 18 \\ 14 \\ \hline r \\ t_5 \end{array}$$

Practice: p. 16/ # 1-6(ac), 7-11, 13, 17, 22, 26