3.3 Solving Right Triangles

Aurora borealis above Churchill, Manitoba

Focus on ...

- explaining the relationships between similar right triangles and the definitions of the trigonometric ratios
- solving right triangles, with or without technology
- solving problems involving one or more right triangles

The polar aurora is one of the most beautiful and impressive displays of nature. There have been various attempts to explain the phenomenon of these northern lights. Carl Stormer, a Norwegian scientist, used a network of cameras that simultaneously photographed the aurora. He used the photos to measure the parallax angle shifts and then calculate the height of the aurora.

Investigate Estimation of Distance

Materials

- metre stick or measuring tape

In this investigation you will use the method of parallax to help you estimate the distance to an object.

1. Have a partner stand a distance away from you. Then, mark the floor where each of you is standing using a small piece of paper or other identifying item, such as masking tape. Stretch out your arm with your thumb pointed upward and close your right eye. Line your thumb up with your partner.

Did You Know?

If you stretch your arm out in front of your face with your thumb pointing upward, and then close one eye, your thumb appears to shift slightly. This shift is known as *parallax*. Your brain uses this information to figure out how far away from you objects are.

2. Open your right eye and close your left eye. Do not move your outstretched arm. Have your partner move to his or her right until he or she is in line with your thumb again. Then, mark the new location where your partner is standing.

3. Use a metre stick to measure the distance from you to your partner and the distance between your partner's locations.

4. **Reflect and Respond**

   a) What is the relationship between the distance to your partner and the distance between your partner's locations? Hint: You may wish to repeat your measurements to help you examine the pattern.

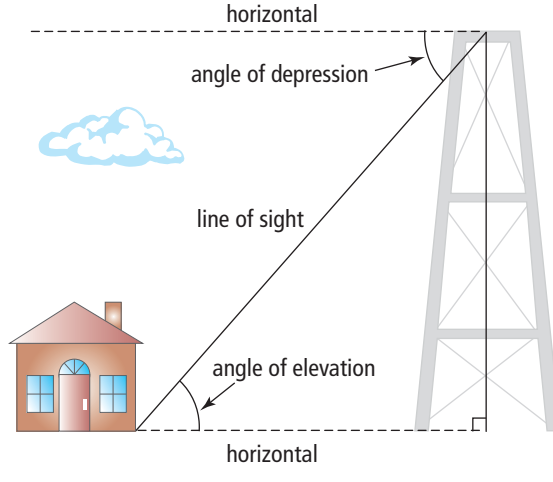
   b) Explain how this relationship can help you estimate your distance to an object.

Link the Ideas

The line of sight is the invisible line from one person or object to another person or object. Some applications of trigonometry involve an angle of elevation and an angle of depression.

- An angle of elevation is the angle formed by the horizontal and a line of sight *above* the horizontal.
- An angle of depression refers to the angle formed by the horizontal and a line of sight *below* the horizontal.

Measure the angle of elevation and the angle of depression in the diagram. How are the measures of two angles related?



Example 1 Use Angle of Elevation to Calculate a Height

Sean wants to calculate the height of the First Nations Native Totem Pole. He positions his transit 19.0 m to the side of the totem pole and records an angle of elevation of 63° to the top of the totem pole. If the height of Sean's transit is 1.7 m, what is the height of the totem pole, to the nearest tenth of a metre?

Solution

Let x represent the height from the transit to the top of the totem pole.

$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan 63° = \frac{x}{19.0}$$

$$x = 19.0(\tan 63°)$$

$$x = 37.289\ldots$$

Height of totem pole
= height of transit + height from transit to top of pole
= 1.7 + 37.289…
= 38.989…

The height of the First Nations Native Totem Pole is 39.0 m, to the nearest tenth of a metre.

Your Turn

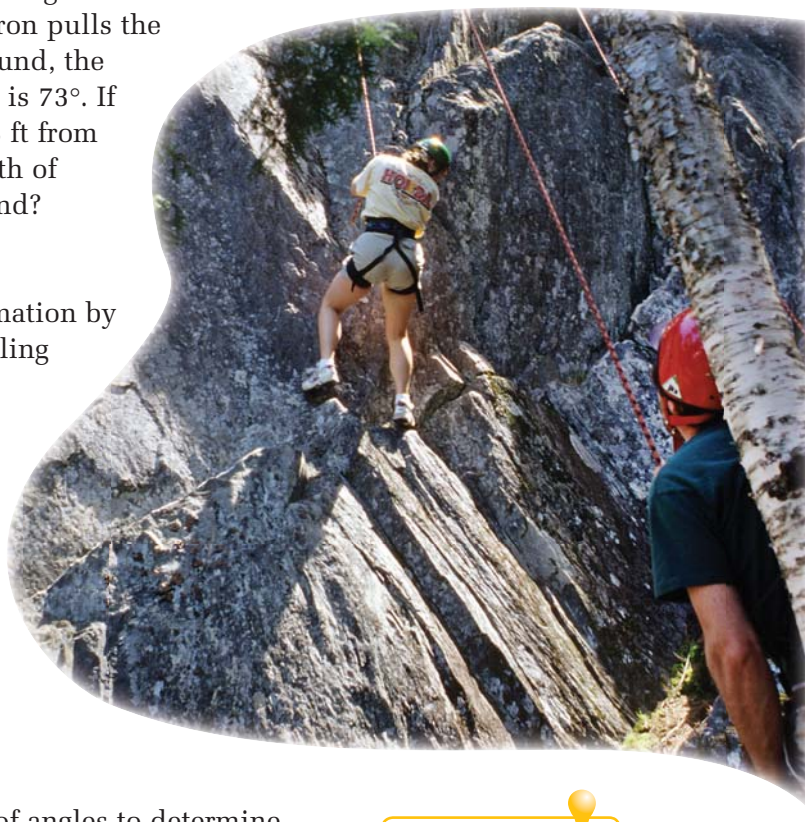
A surveyor needs to determine the height of a large grain silo. He positions his transit 65 m from the silo and records an angle of elevation of 52°. If the height of the transit is 1.7 m, determine the height of the silo, to the nearest metre.

Did You Know?

The First Nations Native Totem Pole is in Beacon Hill Park, in Victoria, BC. The totem pole was erected in 1956 and is one of the world's tallest totem poles.

Example 2 Calculate a Distance Using Angle of Depression

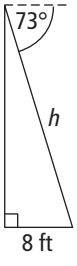
Natalie is rock climbing and Aaron is belaying. When Aaron pulls the rope taut to the ground, the angle of depression is 73°. If Aaron is standing 8 ft from the wall, what length of rope is off the ground?



Solution

Visualize the information by sketching and labelling a diagram.

Let h represent the length of rope that is off the ground.



Use the properties of angles to determine the angle measure of one of the acute angles inside the right triangle.

$\theta = 90° - 73°$
$\theta = 17°$

The angle that the rope makes at the top with the vertical is 17°.

$$\sin 17° = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin 17° = \frac{8}{h}$$

$$h = \frac{8}{\sin 17°}$$

$$h = 27.362\ldots$$

The rope off the ground is approximately 27 ft long.

Your Turn

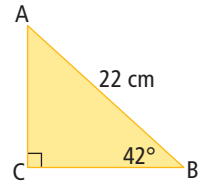
A balloonist decides to use an empty football field for his landing area. When the balloon is directly over the goal post, he measures the angle of depression to the base of the other goal post to be 53.8°. Given that the distance between goal posts in a Canadian football field is 110 yd, determine the height of the balloon.

Did You Know?

A *belayer* is the person on the ground who secures a climber who is rock climbing. The belayer and climber each wear a harness that attaches to a rope. The belayer controls how much slack is in the rope. It takes skill and concentration to be a successful belayer.

Example 3 Solve a Right Triangle

Solve the triangle shown. Express each measurement to the nearest whole unit.



Solution

To *solve* a triangle means to determine the lengths of all unknown sides and the measures of all unknown angles. To solve this triangle, you need to determine the lengths of sides AC and CB and the measure of ∠A.

$\angle A = 180° - (90° + 42°)$ What is the sum of the angles in a triangle?

$\angle A = 48°$

Using ∠B as the reference angle and knowing the length of the hypotenuse, apply the cosine ratio to calculate the length of side CB.

$$\cos B = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos 42° = \frac{CB}{22}$$

$$CB = 22(\cos 42°)$$

$$CB = 16.349\ldots$$

Calculate the length of side AC.

Method 1: Apply a Trigonometric Ratio

Since all angles are known, any of the primary trigonometric ratios could be applied.

$$\sin B = \frac{\text{opposite}}{\text{hypotenuse}}$$ How will you decide which ratio to use?

$$\sin 42° = \frac{AC}{22}$$

$$AC = 22(\sin 42°)$$

$$AC = 14.720\ldots$$

Method 2: Apply the Pythagorean Theorem

$$AB^2 = AC^2 + CB^2$$

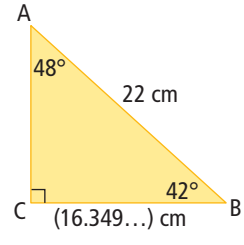
$$22^2 = AC^2 + (16.349\ldots)^2$$

$$484 = AC^2 + 267.295\ldots$$

$$216.704\ldots = AC^2$$

$$\sqrt{216.704\ldots} = AC$$

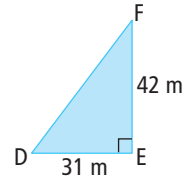
$$14.720\ldots = AC$$



Angle A measures 48°. Side CB is about 16 cm long and side AC is about 15 cm long.

Your Turn

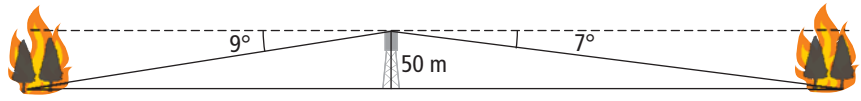
Solve the triangle shown. Express each measurement to the nearest whole unit.



What information are you given? Use the given information as much as possible in your calculations.

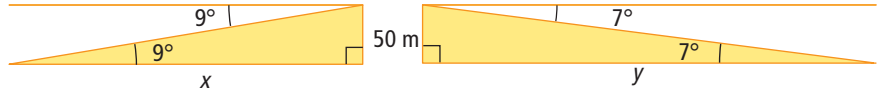
Example 4 Solve a Problem Using Trigonometry

From a height of 50 m in his fire tower near Francois Lake, BC, a ranger observes the beginnings of two fires. One fire is due west at an angle of depression of 9°. The other fire is due east at an angle of depression of 7°. What is the distance between the two fires, to the nearest metre?



Solution

Model the problem using right triangles.
Let x and y represent the lengths of the bases of the triangles.



$$\tan 9° = \frac{\text{opposite}}{\text{adjacent}}$$
$$\tan 9° = \frac{50}{x}$$
$$x = \frac{50}{\tan 9°}$$
$$x = 315.687\ldots$$

Use the given angles to find the measure of one acute angle in each right triangle.

$$\tan 7° = \frac{\text{opposite}}{\text{adjacent}}$$
$$\tan 7° = \frac{50}{y}$$
$$y = \frac{50}{\tan 7°}$$
$$y = 407.217\ldots$$

Add to determine the distance between the fires.
$315.687\ldots + 407.217\ldots = 722.904\ldots$

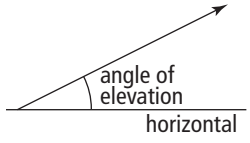
The distance between the fires, to the nearest metre, is 723 m.

Your Turn

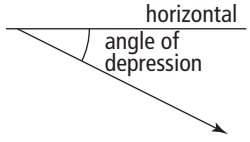
From his hotel window overlooking Saskatchewan Drive in Regina, Ken observes a bus moving away from the hotel. The angle of depression of the bus changes from 46° to 22°. Determine the distance the bus travels, if Ken's window is 100 m above street level. Express your answer to the nearest metre.

Key Ideas

- An angle of elevation is the angle between the line of sight and the horizontal when an observer looks upward.



- An angle of depression is the angle between the line of sight and the horizontal when the observer looks downward.



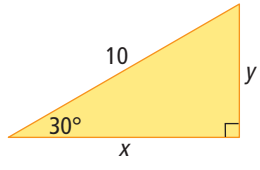
- To solve a triangle means to calculate all unknown angle measures and side lengths.

Check Your Understanding

Practise

1. Solve each triangle, to the nearest tenth of a unit.

a)



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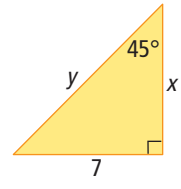
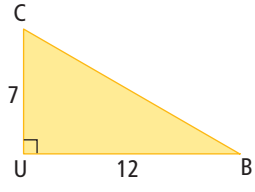
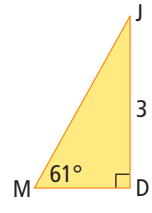


c)

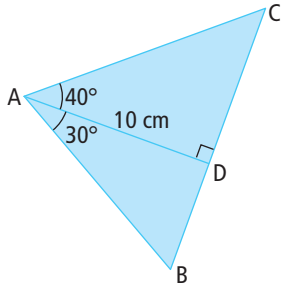


d)

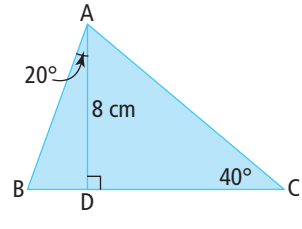


2. Calculate the length of BC, to the nearest tenth of a centimetre.

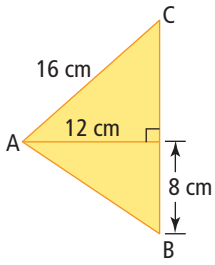
a)



b)



3. Determine the measure of ∠CAB, to the nearest degree.



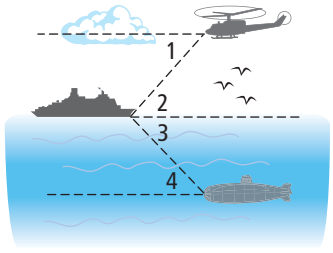
4. Describe each angle as it relates to the diagram.

a) ∠1

b) ∠2

c) ∠3

d) ∠4

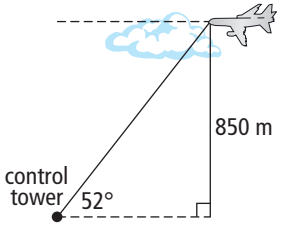


5. The heights of several tourist attractions are given in the table. Determine the angle of elevation from a point 100 ft from the base of each attraction to its top.

	Attraction	Location	Height
a)	World's largest fire hydrant	Elm Creek, MB	29 ft
b)	World's largest dinosaur	Drumheller, AB	80 ft
c)	Saamis Tipi	Medicine Hat, AB	215 ft
d)	World's largest tomahawk	Cut Knife, SK	40 ft
e)	Igloo church	Inuvik, NT	78 ft

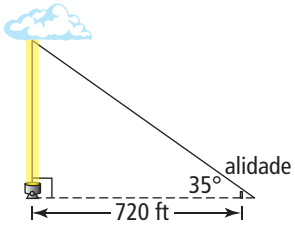
Apply

6. An airplane is observed by an air traffic controller at an angle of elevation of 52°. The airplane is 850 m above the observation deck of the tower. What is the distance from the airplane to the tower? Express your answer to the nearest metre.



7. Cape Beale Lighthouse, BC, is on a cliff that is 51 m above sea level. The lighthouse is there to warn boats of the danger of shallow waters and the possibility of rocks close to the shore. The safe distance for boats from this cliff is 75 m. If the lighthouse keeper is 10 m above ground and observes a boat at an angle of depression of 50°, is the boat a safe distance from the cliff? Justify your conclusion.

8. At night, it is possible to make precise measurements of cloud height using a search light. An alidade is set 720 ft away from the search light. It measures the angle of elevation to the place where the light strikes the cloud to be 35°. What is the altitude of the cloud? Express your answer to the nearest foot.




alidade

9. The working arm of a tower crane is 192 m high and has a length of 71.6 m. Suppose the hook reaches the ground and moves along the arm on a trolley.

a) Determine the maximum distance from the hook to the operator when the trolley is fully extended at 71.6 m and the minimum distance when the trolley is closest to the operator at 8.1 m. Hint: The operator is located at the vertex of the crane.

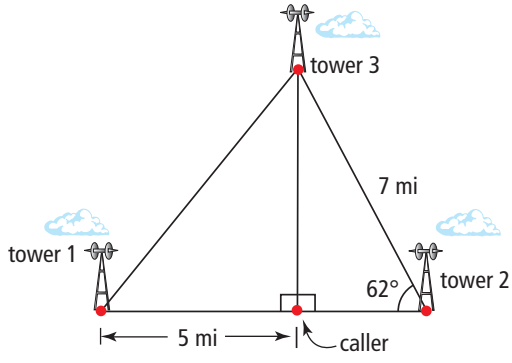
b) Determine the maximum and minimum angles of depression from the operator to the hook on the ground. State your answer to the nearest tenth of a degree.

Did You Know?

For the 2010 Olympic Games in Vancouver, the Millennium Water Project involved building 1100 condominiums. This project made use of eight tower cranes that lifted steel, concrete, large tools, and generators. The cranes often rise hundreds of feet into the air and can reach out just as far.

10. Arctic Wisdom involved children, parents, and Elders gathering on Baffin Island, NU, to send a message. To achieve the best picture of the human image on the sea ice, an aerial photograph was taken. The angle of depression from the helicopter was 58° and the height of the helicopter was 140 m. How far away from the image was the helicopter?

11. **Unit Project** A cell phone can be used to send music, but as your location changes, you move in and out of range from one *cell* to the next. Three or more cellular towers may pick up a cell phone's signal. A cell phone signal has been located 5 mi from tower 1.



a) What is the distance from the caller to tower 3?

b) How far is tower 1 from tower 3?

12. The Disabled Sailing Association had its first sessions at the Jericho Sailing Centre in Vancouver, BC. At a recent regatta, a television news team tracked two sailboats from a helicopter 800 m above the water. The team observed the sailboats on the left and right sides of the helicopter at angles of depression of 58° and 36°, respectively.

a) Which boat is located closer to the helicopter? Explain.

b) Determine the distance between the two boats. Express your answer to the nearest metre.

13. Two tourists stand on either side of the Veterans Pole, honouring Canadian Aboriginal war veterans, in Victoria, BC. One tourist measures the angle of elevation of the top of the pole to be 21°. To the other tourist, the angle of elevation is 17°. If the height of the pole is 5.5 m, how far apart are the tourists? Express your answer to the nearest tenth of a metre.

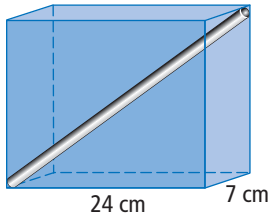
Extend

14. From the top of a 35-m-tall building, an observer sees a truck heading toward the building at an angle of depression of 10°. Ten seconds later, the angle of depression to the truck is 25°.

a) Determine the distance that the truck has travelled. Express your answer to the nearest metre.

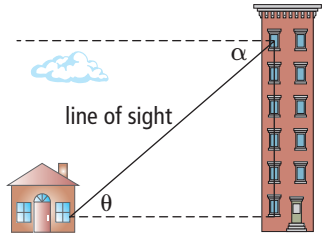
b) If the speed limit for the area is 40 km/h, is the truck driver following the speed limit? Explain.

15. A rectangular prism has base dimensions of 24 cm by 7 cm. A metal rod is run from the bottom corner diagonally to the top corner of the prism. If the rod forms an angle of 40° with the bottom of the box, calculate the volume of the box.



Create Connections

16. From her apartment, Jennie measures the angle of depression to Mike's house. At the same time, Mike measures the angle of elevation to Jennie's apartment.



a) Mike's brother Richard observes Mike and states that Mike made an error, because the angle of elevation must be greater than the angle of depression. Is Richard correct? Explain your reasoning.

b) In order to calculate the measure of angle θ, you can be given any of the following measurements:

- the height of Jennie's window
- the horizontal distance between buildings
- the length of line of sight
- the measure of angle α

Which measurement(s) would you prefer to be given? Explain how you would use these measurements to calculate θ.