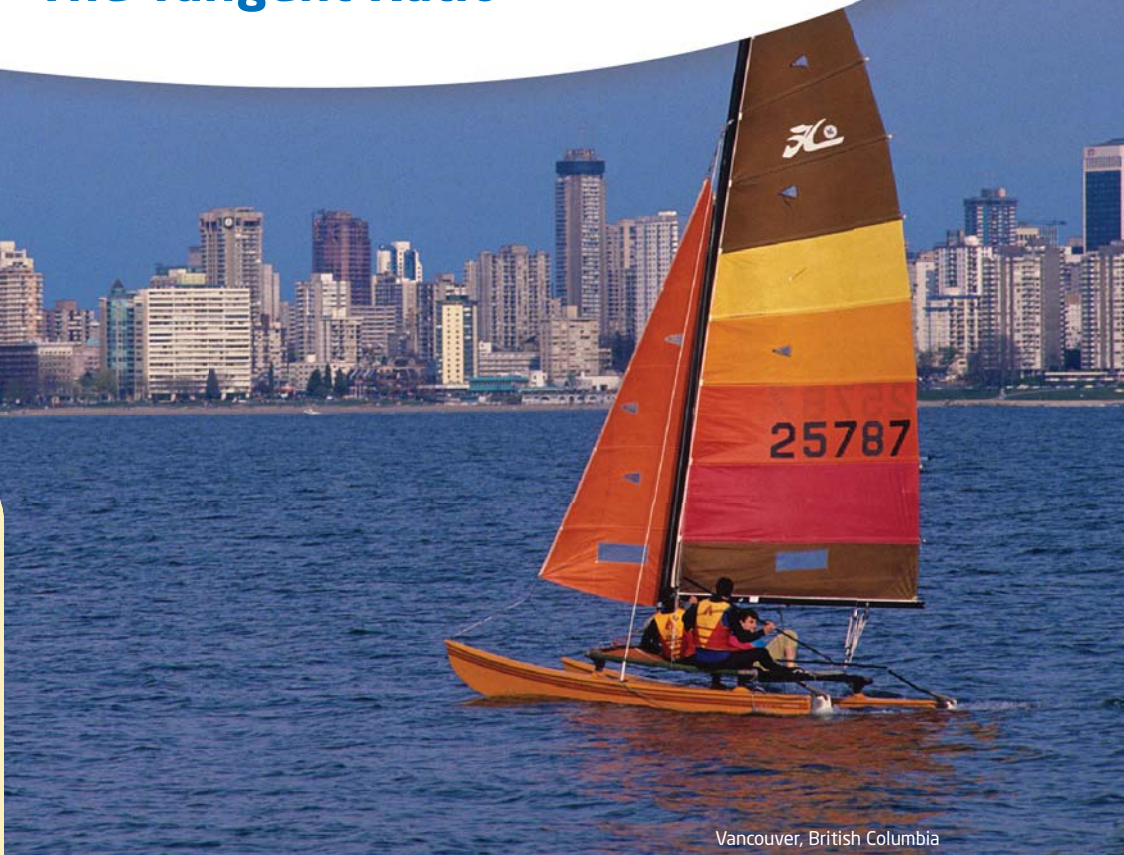


3.1

The Tangent Ratio

Focus on ...

- explaining the relationships between similar triangles and the definition of the tangent ratio
- identifying the hypotenuse, opposite side, and adjacent side for a given acute angle in a right triangle
- developing strategies for solving right triangles
- solving problems using the tangent ratio

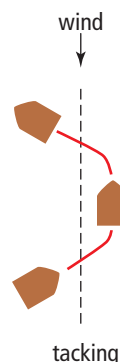


In addition to the Pacific Ocean, there are many lakes in Western Canada that are ideal for sailing. One important aspect of boating is making sure you get where you want to go. Navigation is an area in which trigonometry has played a crucial role; and it was one of the early reasons for developing this branch of mathematics.

People have used applications of trigonometry throughout history. The Egyptians used features of similar triangles in land surveying and when building the pyramids. The Greeks used trigonometry to tell the time of day or period of the year by the position of the various stars. Trigonometry allowed early engineers and builders to measure angles and distances with greater precision. Today, trigonometry has applications in navigating, surveying, designing buildings, studying space, etc.

Investigate the Tangent Ratio

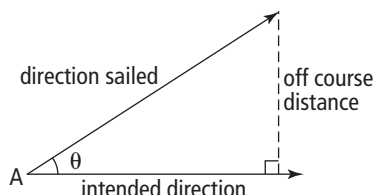
Sailing is a very popular activity. One of the limitations of sailing is that a boat cannot sail directly into the wind. Using a technique called *tacking*, it is possible to sail in almost any direction, regardless of the wind direction. When sailing *on a tack*, you are forced to sail slightly off course and then compensate for the distance sailed when you change direction. You can use trigonometry to determine the distance a boat is off course before changing direction.



Materials

- grid paper
- protractor
- ruler

- On a sheet of grid paper draw a horizontal line 10 cm in length to represent the intended direction.
 - Draw a tacking angle, θ , of 30° .
 - Every two centimetres, along your horizontal line, draw a vertical line to indicate the off course distance. Label the five triangles you created, $\triangle ABC$, $\triangle ADE$, $\triangle AFG$, $\triangle AHI$, and $\triangle AJK$.



- Measure the base and the height for each triangle. Complete the following table to compare the off course distance to the intended direction. In the last column, express the ratio, $\frac{\text{off course distance}}{\text{intended direction}}$, to four decimal places.

Triangle	Intended Direction	Off Course Distance	$\frac{\text{Off Course Distance}}{\text{Intended Direction}}$
$\triangle ABC$			
$\triangle ADE$			
$\triangle AFG$			
$\triangle AHI$			
$\triangle AJK$			

Did You Know?

- vertices of a triangle are commonly labelled with uppercase letters, for example $\triangle ABC$
- angles of a triangle are commonly labelled with Greek letter variables
- some common Greek letters used are theta, θ , alpha, α , and beta, β .

hypotenuse

- the side opposite the right angle in a right triangle

opposite side

- the side across from the acute angle being considered in a right triangle
- the side that does not form one of the arms of the angle being considered

adjacent side

- the side that forms one of the arms of the acute angle being considered in a right triangle, but is not the hypotenuse

3. a) The diagram you drew in step 1c) forms a series of nested similar triangles. How do you know the triangles are similar?

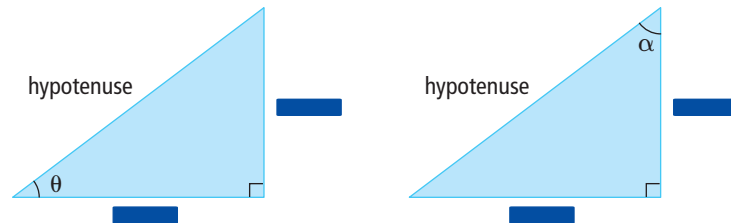
b) Use your knowledge of similar triangles to help describe how changing the side lengths of the triangle affects the ratio $\frac{\text{off course distance}}{\text{intended direction}}$.

4. a) Use your calculator to determine the tangent ratio of 30° . To calculate the tangent ratio of 30° , make sure your calculator is in the degree mode.

Press **C** **TAN** **30** **=**.

b) How does the value on your calculator relate to the data in step 2?

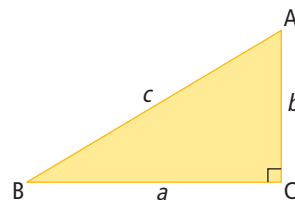
5. In the two right triangles shown, the **hypotenuse** is labelled and an angle is labelled with a variable. Copy each triangle. Use the words **opposite** and **adjacent** to label the side opposite the angle and the side adjacent to the angle.



6. Reflect and Respond

a) Use your results from steps 1 to 4 and the terminology from step 5 to describe a formula you could use to calculate the tangent ratio of any angle.

b) Use your formula to state the tangent ratios for $\angle A$ and $\angle B$ in the following diagram.



Link the Ideas

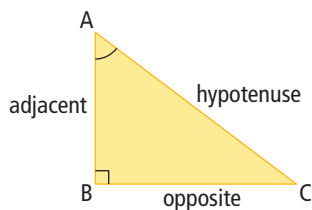
A trigonometric ratio is a ratio of the measures of two sides of a right triangle.

One trigonometric ratio is the

tangent ratio.

The short form for the tangent ratio of angle A is $\tan A$.

$$\tan A = \frac{\text{length of side opposite } \angle A}{\text{length of side adjacent to } \angle A}$$



tangent ratio

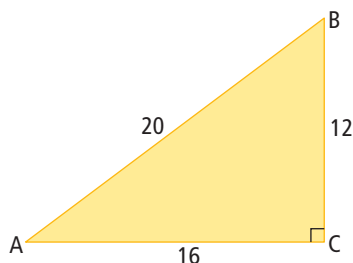
- for an acute angle in a right triangle, the ratio of the length of the opposite side to the length of the adjacent side adjacent
- $\tan A = \frac{\text{opposite}}{\text{adjacent}}$

Example 1 Write a Tangent Ratio

Write each trigonometric ratio.

a) $\tan A$

b) $\tan B$



Solution

a) $\tan A = \frac{\text{opposite}}{\text{adjacent}}$

$$\tan A = \frac{BC}{AC}$$

$$\tan A = \frac{12}{16}$$

$$\tan A = \frac{3}{4}$$

b) $\tan B = \frac{\text{opposite}}{\text{adjacent}}$

$$\tan B = \frac{AC}{BC}$$

$$\tan B = \frac{16}{12}$$

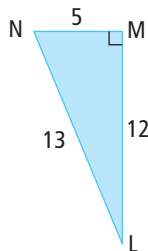
$$\tan B = \frac{4}{3}$$

Your Turn

Calculate each trigonometric ratio.

a) $\tan L$

b) $\tan N$

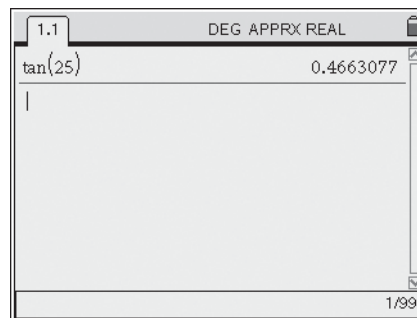


Example 2 Calculate a Tangent and an Angle

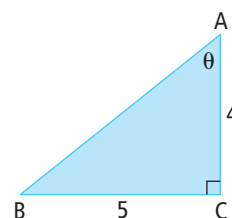
- Calculate $\tan 25^\circ$ to four decimal places.
- Draw a triangle to represent $\tan \theta = \frac{5}{4}$. Calculate the angle θ to the nearest tenth of a degree.

Solution

a) $\tan 25^\circ \approx 0.4663$



- b) Since $\tan \theta = \frac{5}{4}$, the side opposite the angle θ is labelled 5 and the side adjacent to the angle θ is labelled 4. The inverse function on a calculator allows you to apply the tangent ratio in reverse. If you know the ratio, you can calculate the angle whose tangent this ratio represents.

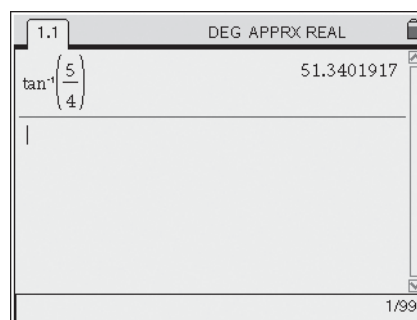


$$\tan \theta = \frac{5}{4}$$

$$\theta = \tan^{-1}\left(\frac{5}{4}\right)$$

$$\theta = 51.340\dots^\circ$$

The angle θ is 51.3° , to the nearest tenth of a degree.



Your Turn

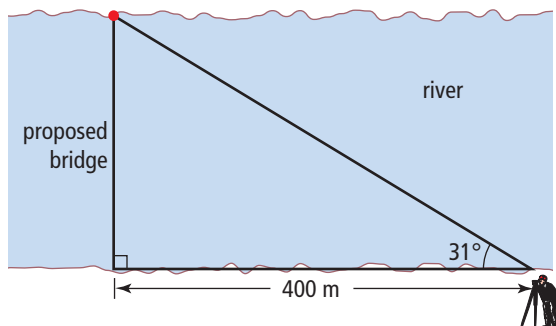
Explore your particular calculator to determine the sequence of keys required. Then, calculate each tangent ratio and angle.

θ	Tan θ
27°	
45°	
57°	

θ	Tan θ
	0.5095
	0.5543
	1.4653

Example 3 Determine a Distance Using the Tangent Ratio

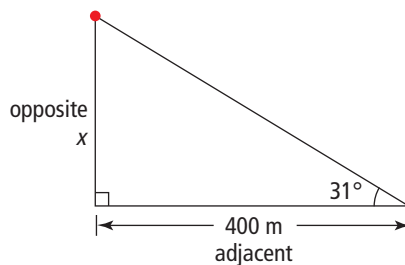
A surveyor wants to determine the width of a river for a proposed bridge. The distance from the surveyor to the proposed bridge site is 400 m. The surveyor uses a theodolite to measure angles. The surveyor measures a 31° angle to the bridge site across the river. What is the width of the river, to the nearest metre?



Solution

Let x represent the distance across the river.

Identify the sides of the triangle in reference to the given angle of 31° .



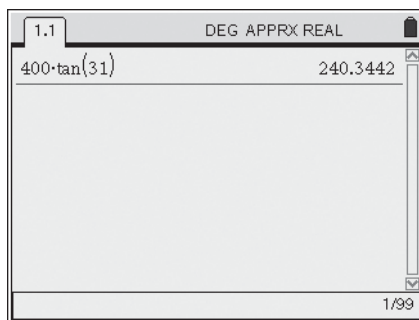
$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan 31^\circ = \frac{x}{400}$$

$$400(\tan 31^\circ) = x$$

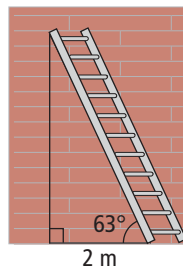
$$240.344... = x$$

To the nearest metre, the width of the river is 240 m.



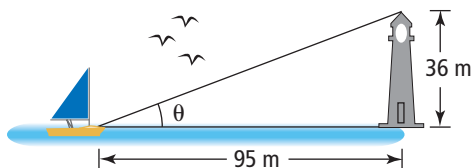
Your Turn

A ladder leaning against a wall forms an angle of 63° with the ground. How far up the wall will the ladder reach if the foot of the ladder is 2 m from the wall?



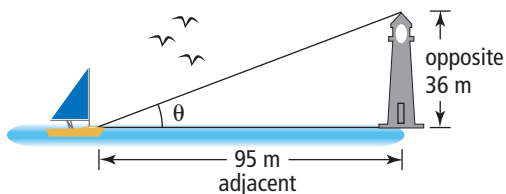
Example 4 Determine an Angle Using the Tangent Ratio

A small boat is 95 m from the base of a lighthouse that has a height of 36 m above sea level. Calculate the angle from the boat to the top of the lighthouse. Express your answer to the nearest degree.



Solution

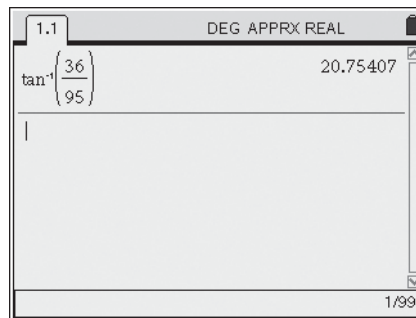
Identify the sides of the triangle in reference to the angle of θ .



$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan \theta = \frac{36}{95}$$

$$\theta = 20.754\dots$$



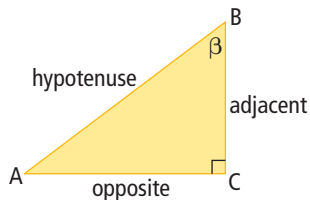
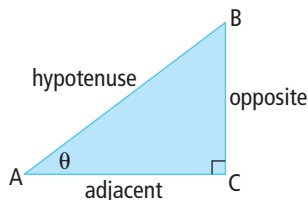
The angle from the boat to the top of the lighthouse is approximately 21° .

Your Turn

A radio transmission tower is to be supported by a guy wire. The wire reaches 30 m up the tower and is attached to the ground a horizontal distance of 14 m from the base of the tower. What angle does the guy wire form with the ground, to the nearest degree?

Key Ideas

- In similar triangles, corresponding angles are equal, and corresponding sides are in proportion. Therefore, the ratios of the lengths of corresponding sides are equal.
- The sides of a right triangle are labelled according to a reference angle.



- The tangent ratio compares the length of the side opposite the reference angle to the length of the side adjacent to the angle in a right triangle.

$$\tan \theta = \frac{\text{length of side opposite } \theta}{\text{length of side adjacent to } \theta}$$

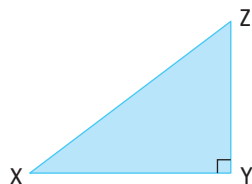
- You can use the tangent ratio to
 - determine the measure of one of the acute angles when the lengths of both legs in a right triangle are known
 - determine a side length if the measure of one acute angle and the length of one leg of a right triangle are known

Check Your Understanding

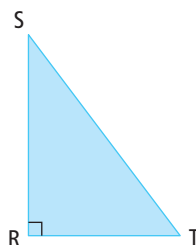
Practise

1. Identify the hypotenuse, opposite, and adjacent sides associated with each specified angle.

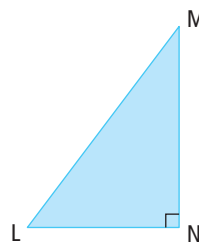
a) $\angle X$



b) $\angle T$



c) $\angle L$



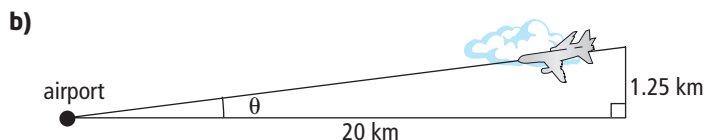
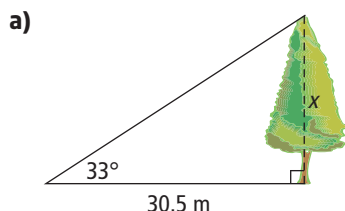
2. Draw right $\triangle DEF$ in which $\angle F$ is the right angle.
 - a) Label the leg opposite $\angle D$ and the leg adjacent to $\angle D$.
 - b) State the tangent ratio of $\angle D$.



Did You Know?

The Franco-Albertan flag was created by Jean-Pierre Grenier. The flag was adopted by the Association canadienne-française de l'Alberta in March 1982.

3. Determine each tangent ratio to four decimal places using a calculator.
 - a) $\tan 74^\circ$
 - b) $\tan 45^\circ$
 - c) $\tan 60^\circ$
 - d) $\tan 89^\circ$
 - e) $\tan 37^\circ$
 - f) $\tan 18^\circ$
4. Determine the measure of each angle, to the nearest degree.
 - a) $\tan A = 0.7$
 - b) $\tan \theta = 1.75$
 - c) $\tan \beta = 0.5543$
 - d) $\tan C = 1.1504$
5. Draw and label a right triangle to illustrate each tangent ratio. Then, calculate the measure of each angle, to the nearest degree.
 - a) $\tan \alpha = \frac{2}{3}$
 - b) $\tan B = \frac{5}{2}$
6. Determine the value of each variable. Express your answer to the nearest tenth of a unit.



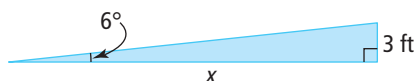
7. Kyle Shewfelt, from Calgary, AB, was the Olympic floor exercise champion in Athens in 2004. Gymnasts perform their routines on a 40-ft by 40-ft mat. They use the diagonal of the mat because it gives them greater distance to complete their routine.
 - a) Use the tangent ratio to determine the angle of the gymnastics run relative to the sides of the mat.
 - b) To the nearest foot, how much longer is the diagonal of the mat than one of its sides?

Apply

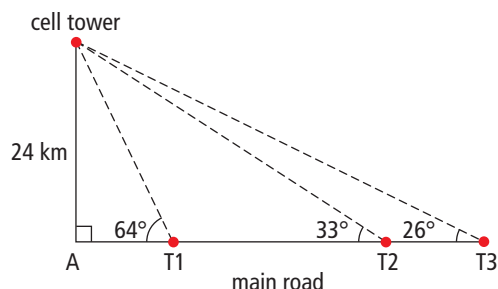
8. Claudette wants to calculate the angles of the triangle containing the fleur-de-lys on the Franco-Albertan flag. She measures the legs of the triangle to be 154 cm and 103 cm. What are the angle measures?



9. A ramp enables wheelchair users and people pushing wheeled objects to more easily access a building.



- a) Determine the horizontal length, x , of the ramp shown. State your answer to the nearest foot.
- b) For a safe ramp, the ratio of vertical distance:horizontal distance needs to be less than 1:12. Would the ramp shown be considered a safe ramp? Explain.
10. **Unit Project** A satellite radio cell tower provides signals to three substations, T1, T2, and T3. The three substations are each located along a stretch of the main road. The cell tower is located 24 km down a road perpendicular to the main road. A surveyor calculates the angle from T1 to the cell tower to be 64° , from T2 to the cell tower to be 33° , and from T3 to the cell tower to be 26° . Calculate the distance of each substation from the intersection of the two roads. Express your answers to the nearest tenth of a kilometre.



11. In the construction of a guitar, it is important to consider the tapering of the strings and neck. The tapering affects the tone that the strings make. For the *Six String Nation Guitar* shown, suppose the width of the neck is tapered from 56 mm to 44 mm over a length of 650 mm. What is the angle of the taper for one side of the guitar strings?



Did You Know?

The *Six String Nation Guitar*, nicknamed *Voyageur*, is made from 63 pieces of history and heritage, from every part of Canada. It represents many different cultures, communities, and characters. The guitar is made from pieces of wood, bone, steel, shell, and stone from every province and territory. It literally embodies Canadian history.

- 12.** When approaching a runway, a pilot needs to maneuver the aircraft, so that it can approach the runway at a constant angle of 3° . A pilot landing at Edmonton International Airport begins the final approach 30 380 ft from the end of the runway. At what altitude should the aircraft be when beginning the final approach? State your answer to the nearest foot.
- 13.** The Idaà Trail is a traditional route of the Dogrib, an Athapaskan-speaking group of Dene. It stretches from Great Bear Lake to Great Slave Lake, in the Northwest Territories. Suppose a hill on the trail climbs 148 ft vertically over a horizontal distance of 214 ft.
- Calculate the angle of steepness of the hill.
 - How far would you have to climb to get to the top of the hill?

Extend

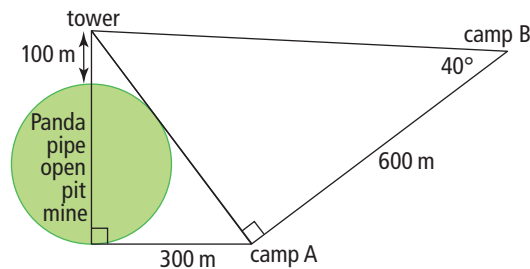
Did You Know?

Ekati mine is Canada's first diamond mine. It is located 200 km south of the Arctic Circle in the Northwest Territories. Diamond mines contain pipes, which are cylindrical pits where diamonds are found.

- 14.** One of the Ekati mine's pipes, called the Panda pipe, has northern and southern gates. A communications tower stands 100 m outside the north gate. The tower can be seen from a point 300 m east of the south gate at camp A.
- The distance between camp A and camp B is 600 m. Calculate the diameter of the Panda pipe.
 - Calculate the distance from camp B to the tower.

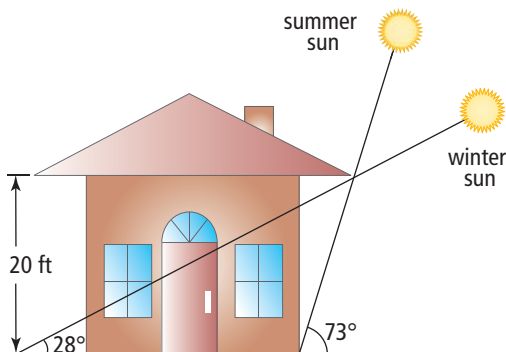


Panda Pipe



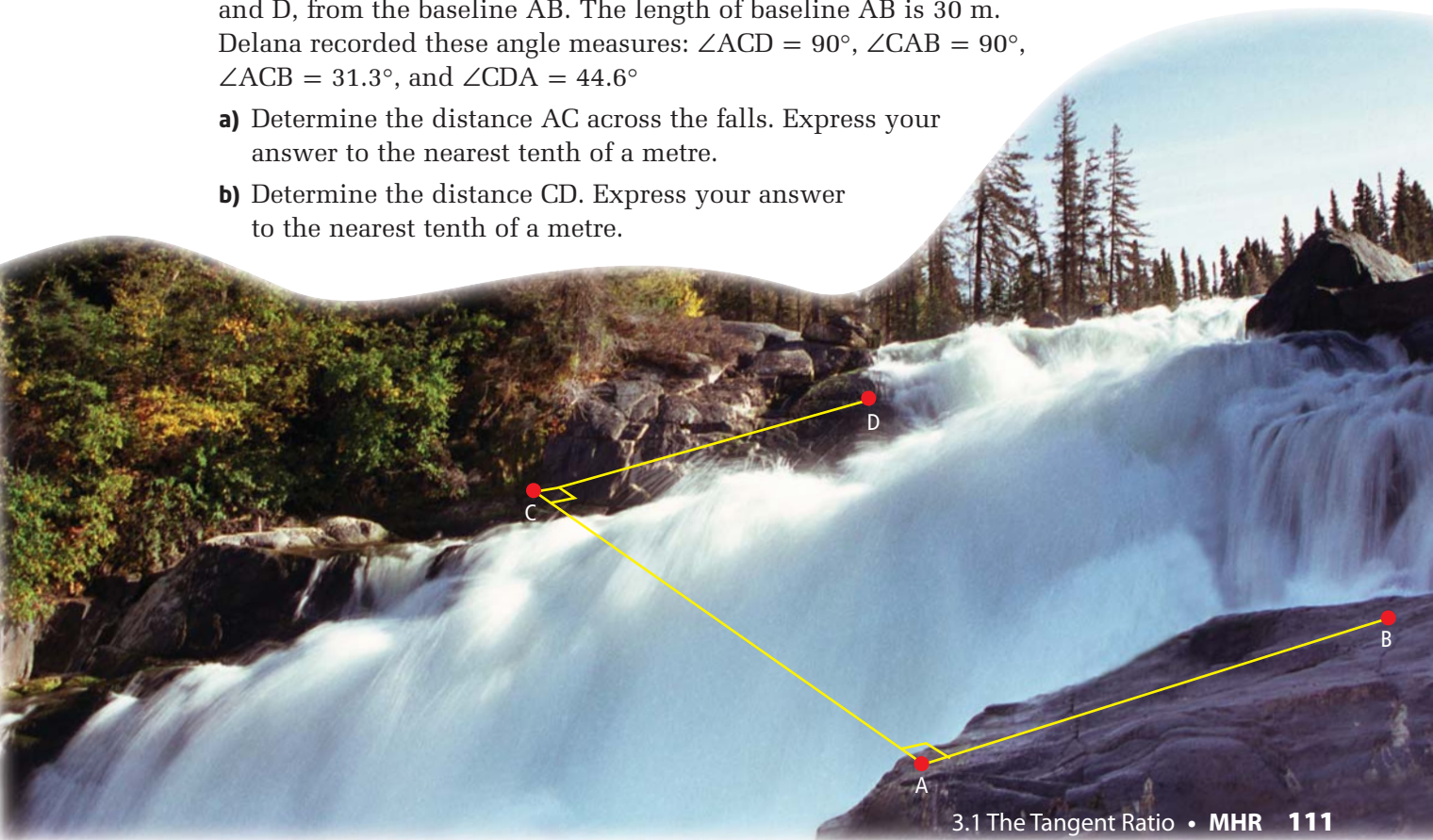
15. Habitat for Humanity Saskatoon has designed a home that provides passive solar features. The idea is to keep the sun off the outside south wall during the summer months and to have the wall exposed to the sun as much as possible during the winter months. The highest angle of the sun during the summer months is 73° .

- a) Suppose the wall of the house is 20 ft tall. How much overhang on the roof trusses should be provided so that the shadow of the noonday sun reaches the bottom of the wall during the summer months?
- b) The lowest angle of the sun during the winter months is 28° . What height of the wall will be in direct sunlight during the winter months?



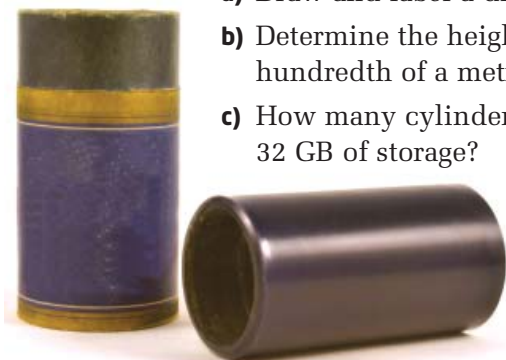
16. Nistowiak Falls, located in Lac LaRonge Provincial Park is one of the highest waterfalls in Saskatchewan. Delana, a surveyor, needs to measure the distance across the falls. She sighted two points, C and D, from the baseline AB. The length of baseline AB is 30 m. Delana recorded these angle measures: $\angle ACD = 90^\circ$, $\angle CAB = 90^\circ$, $\angle ACB = 31.3^\circ$, and $\angle CDA = 44.6^\circ$

- a) Determine the distance AC across the falls. Express your answer to the nearest tenth of a metre.
- b) Determine the distance CD. Express your answer to the nearest tenth of a metre.



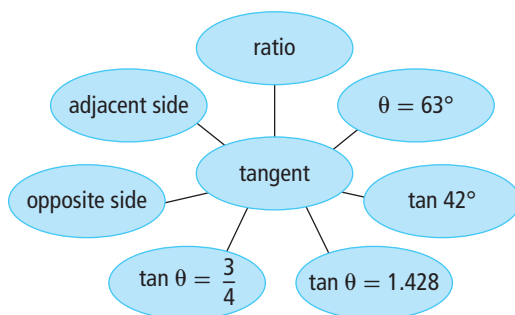
- 17. (Unit Project)** The first sound recordings were done on wax cylinders that were 5 cm in diameter and 10 cm long. Wax cylinders were capable of recording about 2 min of sound. Modern music storage devices can have tremendous memory and store thousands of songs. Janine calculated the number of wax cylinders needed to match a 32 GB storage capacity. Imagine that these cylinders are stacked one on top of another. From a distance of 10 m, the angle of elevation to the top of the stack would be 89.5° .

- Draw and label a diagram to represent the situation.
- Determine the height of the stack of cylinders, to the nearest hundredth of a metre.
- How many cylinders would need to be stacked to match 32 GB of storage?

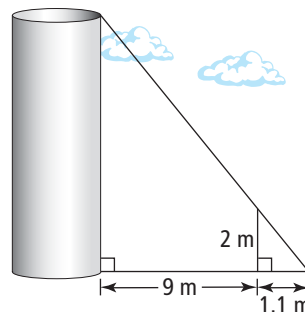


Create Connections

- 18.** Copy the following graphic organizer. For each item, describe its meaning and how it relates to the tangent ratio.

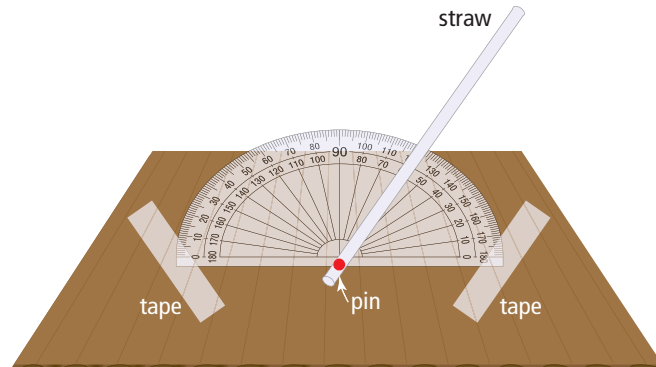


- 19.** Draw a right triangle in which the tangent ratio of one of the acute angles is 1. Describe the triangle.
- 20.** Devin stores grain in a cylindrical granary. Suppose Devin places a 2-m-tall board 9 m from the granary and 1.1 m away from a point on the ground. Describe how Devin could use trigonometry to calculate the angle formed with the ground and the top of the granary. Then, determine this angle.



21. **MINI LAB** When measuring inaccessible distances, a surveyor can take direct measurements using a transit. A transit can measure both horizontal and vertical angles.

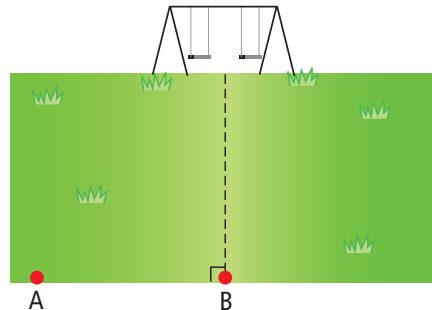
Step 1 Construct a transit as shown in the diagram. Pin the straw at the centre of the protractor.



Materials

- piece of cardboard
- large protractor
- drinking straw
- tape
- pin
- measuring tape

Step 2 Explain how a transit could be used to assess the distance to an object. Hint: You will need to draw and measure a baseline. This is the line from A to B in the diagram.



Step 3 To calculate the distance to some objects in your schoolyard, use your transit to measure the required angles.

Object	Length of Baseline AB	Measure of $\angle A$	Distance to the Object