7.1 Slope-Intercept Form

Focus on ...

- identifying the slope and *y*-intercept of a straight-line graph
- determining a linear equation using slope and *y*-intercept
- rewriting a linear relation in slope-intercept form
- graphing equations in slope-intercept form
- solving problems using equations in slope-intercept form

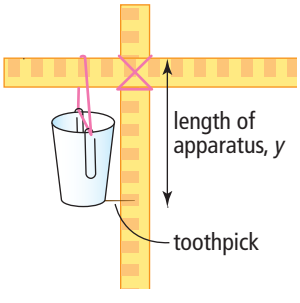
Many relationships can be modelled by the graph of a straight line. For example, a farmer purchases chicken feed pellets by mass, yet dispenses the feed by volume. When the farmer analyses the feeding of chickens, the relationship between the volume of feed pellets in a dispenser and the mass of the feed may be modelled by a linear equation. Linear equations can be written in different forms. Each form can provide specific information about the graph.

Investigate the Graph of a Linear Equation

Materials

- two metre sticks
- elastic band
- foam cup
- paper clips, string, or tape
- toothpick or straightened paper clip
- six identical marbles or other items of equal mass
- ruler
- grid paper

1. a) Suspend a metre stick between two chairs or desks. Attach an elastic band across the diameter of a foam cup using paper clips. Loop the other end of the elastic band around the metre stick. Poke a toothpick horizontally through the bottom edge of the cup.



b) Position another metre stick vertically beside the suspended cup. Measure the length of the apparatus, *y*, in centimetres, from the top of the elastic band to the bottom of the cup. Use the toothpick to help you locate the reading on the metre stick.

2. a) Place one marble in the cup. Let the cup come to rest. Measure and record the length of the apparatus. One at a time, place the other marbles in the cup. After adding each marble, measure the length of the apparatus again.

b) On grid paper, plot the individual data points. Plot the number of marbles along the x-axis. Using a ruler, draw a straight line that represents the data.

3. Identify the independent and dependent variables.

4. Determine the slope of the line. What are the units of the slope? What does the slope of the line represent in this situation?

5. a) Identify the point where the line intersects the y-axis. What does this point represent in this situation?

b) Starting at this point, explain how to use the slope to determine the length of the apparatus when one, two, or three marbles are in the cup.

6. Use the ***y*-intercept** and slope to determine what the length of the apparatus would be if ten marbles were in the cup. Does your graph support your answer?

7. a) Write the equation of the line in **slope-intercept form**. Let y represent the length of the apparatus, in centimetres. Let x represent the number of marbles in the cup.

b) Use the equation to estimate the length of the apparatus if the cup contains 15 marbles.

8. Reflect and Respond Suppose you use a cup that is 4 cm taller but has the same mass as the foam cup used.

a) How would this change the data you collected?

b) Describe how the graph would change.

c) Write an equation in slope-intercept form that represents this situation.

9. Suppose you use items that are double the mass of the marbles used.

a) How would this change your data?

b) Describe how the graph would change.

c) Represent this situation using an equation written in slope-intercept form.

***y*-intercept**

- the y-coordinate of the point where a line or curve crosses the y-axis
- the value of y when $x = 0$

slope-intercept form

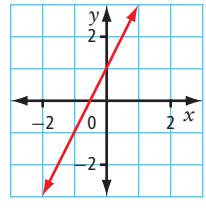
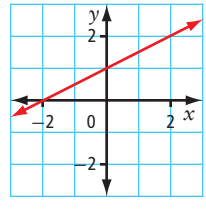
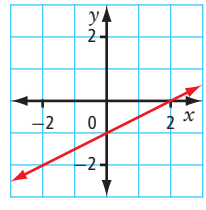
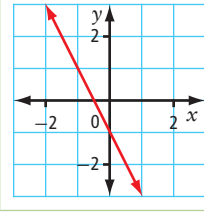
- the equation of a line in the form $y = mx + b$, where m is the slope of the line and b is the y-intercept

Link the Ideas

To write the equation of a straight-line graph, you can use the following two constants:

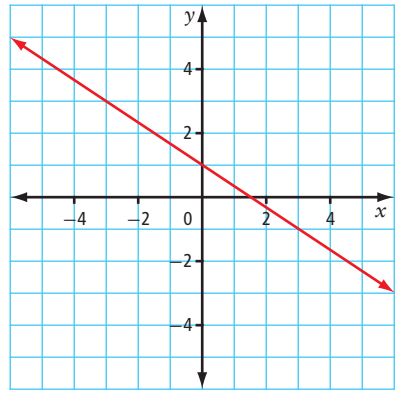
- the rate of change or slope, m
- the y-intercept. If $(0, b)$ is the point where the line crosses the y-axis, then b is the y-intercept.

The equation of a non-vertical straight-line graph can be written in slope-intercept form. The equation is $y = mx + b$, where m represents the slope $\left(\frac{\text{rise}}{\text{run}}\right)$ and b represents the y-intercept.

Table of Values	Graph	Slope, m	y-intercept, b	Equation, $y = mx + b$
x | y 0 | 1 1 | 3 2 | 5		$m = \frac{\Delta y}{\Delta x}$ $m = \frac{2}{1}$ $m = 2$	1	$y = 2x + 1$
x | y 0 | 1 2 | 2 4 | 3		$m = \frac{\Delta y}{\Delta x}$ $m = \frac{1}{2}$	1	$y = \frac{1}{2}x + 1$
x | y 0 | −1 2 | 0 4 | 1		$m = \frac{\Delta y}{\Delta x}$ $m = \frac{1}{2}$	−1	$y = \frac{1}{2}x + (-1)$
x | y 0 | −1 1 | −3 2 | −5		$m = \frac{\Delta y}{\Delta x}$ $m = \frac{-2}{1}$ $m = -2$	−1	$y = -2x + (-1)$

Example 1 Write the Equation of a Line in Slope-Intercept Form

a) What are the slope and y-intercept of the line shown in the graph?
b) Write the equation of the line in slope-intercept form, $y = mx + b$.
c) Use graphing technology to check your equation.



Solution

a) The y-intercept is 1. Therefore, $b = 1$.
Using the points (0, 1) and (3, −1), the slope is

$$m = \frac{\text{rise}}{\text{run}}$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{-1 - 1}{3 - 0}$$

$$m = -\frac{2}{3}$$

The slope is $-\frac{2}{3}$ and the y-intercept is 1.

What do you know about the slope if the line falls from left to right?

How else could you determine the slope?

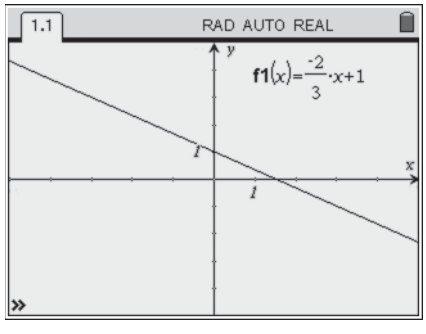
b) Substitute the values of m and b into the slope-intercept form of an equation.

$$y = mx + b$$

$$y = -\frac{2}{3}x + 1$$

The equation of the line in slope-intercept form is $y = -\frac{2}{3}x + 1$.

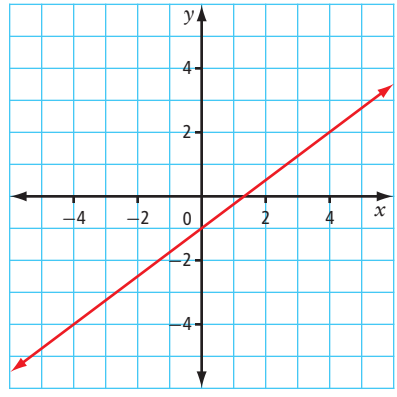
c) Create the graph using technology.



How can you confirm that this is the equation of the line that passes through the points (0, 1) and (3, −1)?

Your Turn

a) What are the slope and y-intercept of the line shown in the graph?



b) What is the equation of the line in slope-intercept form, $y = mx + b$?

c) Use graphing technology to check your equation.

Example 2 Convert an Equation to Slope-Intercept Form

A students' council rents a portable dunk tank as a fund-raising activity. Students pay for the chance to hit a target with a ball and dunk a teacher into a tank of water.

The relationship between the number of balls thrown, x, and the profit, y, in dollars, may be represented by the equation $3x - 2y - 600 = 0$.

a) Rewrite the equation in slope-intercept form.

b) State the slope of the line. What does the slope represent?

c) Identify the y-intercept. What does it represent?

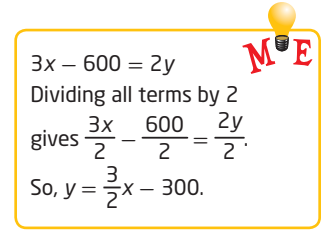
d) The break-even point is the point at which the money raised equals the money spent. How many balls must the students sell to reach the break-even point?

Solution

a) Rearrange the equation into the form $y = mx + b$. To do this, isolate the variable y.

$$\begin{aligned} 3x - 2y - 600 &= 0 \\ 3x - 2y - 600 + 2y &= 0 + 2y \\ 3x - 600 &= 2y \\ \frac{3}{2}x - 300 &= y \end{aligned}$$

The equation written in slope-intercept form is $y = \frac{3}{2}x - 300$.



b) The slope of the line is $\frac{3}{2}$ or 1.5. It represents income of \$1.50 per ball. The slope is positive because the money is coming in.

c) The y-intercept is -300. It represents a cost of \$300 to rent the portable dunk tank. It is negative because the money is paid out as an expense.

d) At the break-even point, students do not make or lose money. So, the profit is zero.

Substitute $y = 0$ into the equation and solve for x.

$$\begin{aligned} y &= \frac{3}{2}x - 300 \\ 0 &= \frac{3}{2}x - 300 \\ 300 &= \frac{3}{2}x - 300 + 300 \\ 2(300) &= \overset{1}{\cancel{2}}\left(\frac{3}{\underset{1}{\cancel{2}}}x\right) \\ 600 &= 3x \\ 200 &= x \end{aligned}$$

To reach the break-even point, they must sell 200 balls, at a rate of \$1.50 per ball. They will make money if they sell more than 200 balls. They lose money if they sell fewer than 200 balls.

Your Turn

Parents of members of the cheerleading squad rent a hall. They arrange a talent show as a fundraiser. The relationship between the number of tickets sold, x, and the profit, y, in dollars, may be represented by the equation $12x - y - 840 = 0$.

a) What is the slope of the line? What does the slope represent?

b) Identify the y-intercept. What does it represent?

c) How many tickets must the parents sell to reach the break-even point?

Did You Know?

In 1998, the Canadian Armed Forces purchased four submarines from Britain. The submarines were named after Canadian port cities. The *Victoria* is based at Esquimalt, BC. It is part of the Maritime Forces Pacific fleet.

parameter

- a variable that has a constant value in a particular equation

Example 3 Model and Solve a Problem Using an Equation in Slope-Intercept Form

Submarines must withstand tremendous pressure exerted on all sides by the water. The table shows the linear relationship between pressure and water depth.

Depth (m)	Pressure (kPa)
0	100
25	350
50	600
75	850

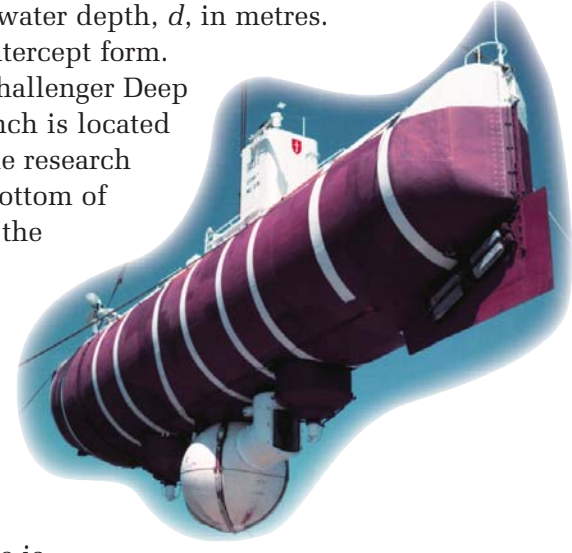
a) Sketch a graph of the data.

b) What is the slope of the line? What does it represent?

c) Determine the value of the **parameter** b. What does this value represent?

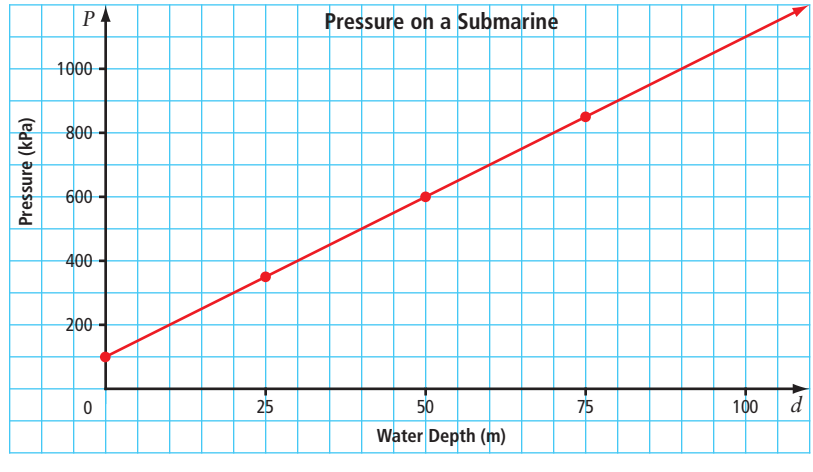
d) Write an equation that models the relationship between pressure, P, in kilopascals, and water depth, d, in metres. Express the equation in slope-intercept form.

e) The deepest point on Earth is Challenger Deep in the Mariana Trench. This trench is located in the Pacific Ocean. In 1960, the research submarine *Trieste* reached the bottom of Challenger Deep. At this depth, the walls protecting the two crew members had to withstand a pressure of 109 300 kPa. What is the approximate depth of Challenger Deep?



Solution

a) The independent variable is depth. The dependent variable is pressure. Plot the coordinate pairs (0, 100), (25, 350), (50, 600), and (75, 850). Both depth and pressure can be any real number, so join the points with a straight line.



b) Determine the ratio of vertical change to horizontal change.

$$m = \frac{\Delta y}{\Delta x}$$
$$m = \frac{250}{25}$$
$$m = 10$$

What values could you use to calculate Δy and Δx?

The slope of the line is 10. This means that for every metre you descend, the pressure increases by 10 kPa.

c) The parameter, b, represents the y-intercept, which is equal to 100. The air pressure is 100 kPa at the surface of the water, where water depth is 0 m.

d) Substitute $m = 10$ and $b = 100$ into $y = mx + b$.

$y = 10x + 100$

If P represents pressure and d represents depth, then the equation of the line is $P = 10d + 100$.

e) Substitute 109 300 for P.

$$P = 10d + 100$$
$$109\,300 = 10d + 100$$
$$109\,200 = 10d$$
$$10\,920 = d$$

The approximate depth of Challenger Deep is 10 920 m.

Your Turn

Asha has selected a hotel for her wedding reception. The cost involves a fee for the deluxe ballroom and a buffet charge that depends on the number of guests. This is shown in the table.

Number of Guests	Cost ($)
0	425
25	1800
50	3175
100	5925

a) Sketch a graph of the data in the table.
b) What are the slope and y-intercept of the line? What does each parameter represent?
c) Write an equation that describes the relationship between the cost and the number of guests. Express the equation in slope-intercept form.
d) What is the cost for 140 guests?
e) Asha would like the total cost to be no more than $15 000. What is the maximum number of guests that can attend?

Did You Know?

To boil water, some First Nations people used to dig a bowl-shaped pit in the ground. Then, they lined the bottom with buffalo hide. They added water and red-hot rocks to the pit, until the water boiled.

Example 4 Determine an Unknown Parameter

An archaeologist simulates a First Nations method of boiling water by adding hot rocks to an earthen pit filled with water. As the rocks cool and lose their heat, the archaeologist replaces them with new rocks from the fire. Suppose the water temperature rises at a constant rate. The temperature of the water at the start of the experiment is 10 °C.

Cooking in a Fire Pit by Shayne Tolman
Painting on display at Head-Smashed-In Buffalo Jump Interpretive Centre likely represents 2500-year-old Besant culture.

The equation $W = mt + 10$ models how the temperature of the water, W, in degrees Celsius, increases at a constant rate of m degrees Celsius per minute for t minutes.

a) After 5 min, the water temperature is 19 °C. Determine the value of the parameter m. What does m represent?

b) How long will it take for the water to boil?

Solution

a) Represent a temperature of 19 °C after 5 min as the point (5, 19). Substitute the coordinates of the point (5, 19) into the given equation. Solve for m.

$$W = mt + 10$$
$$19 = m(5) + 10$$
$$19 - 10 = 5m$$
$$9 = 5m$$
$$\frac{9}{5} = m \text{ or } m = 1.8$$

The parameter m represents the rate at which the temperature of the water increases per minute. The water temperature increases at a rate of 1.8 °C/min.

b) Solve the equation for t when W is 100 °C.

What does $W = 100$ °C represent?

$$W = 1.8t + 10$$
$$100 = 1.8t + 10$$
$$90 = 1.8t$$
$$50 = t$$

The water boils after 50 min.

Did You Know?

The coordinates of a point on a graph satisfy the equation of the graph. Conversely, a point whose coordinates satisfy an equation is a point on a graph of the equation.

For example,
The point (1, 8) is a point on the graph of $y = 3x + 5$.
Substituting $x = 1$ and $y = 8$.
$8 = 3(1) + 5$
$8 = 8$ True.

Conversely, since the point (1, 8) satisfies the equation, (1, 8) is a point on the graph of the equation $y = 3x + 5$.

Your Turn

A decorator's fee can be modelled by the equation $F = 75t + b$. In the equation, F represents the fee, in dollars, t represents time, in hours, and b represents the cost of the initial consultation, in dollars.

a) Suppose the decorator spends 4 h working for a client and charges the client $450. Determine the value of the parameter b.

b) How many hours does the decorator work if a client is charged $975?

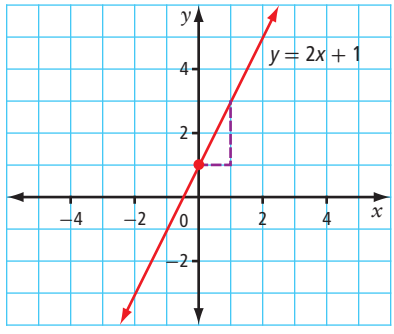
Key Ideas

- The slope-intercept form of a linear equation is $y = mx + b$, where m represents the slope and b represents the y-intercept.

$y = 2x + 1$

slope $= 2$ $\qquad$ y-intercept $= 1$

$\frac{\text{rise}}{\text{run}} = \frac{2}{1}$ $\qquad$ The graph passes through (0, 1).



Check Your Understanding

Practise

1. What are the slope and y-intercept of each line?

a) $y = -5x + 4$ $\qquad$ **b)** $y = \frac{3}{4}x + 1$

c) $y = x - 7$ $\qquad$ **d)** $y = -4x$

e) $y = -3$ $\qquad$ **f)** $y = 0.5x - 1.25$

2. Consider the line $y = -3x + 2$.

a) What are the slope and y-intercept of the line?

b) Explain how you could sketch the graph of this line using the slope and y-intercept.

3. Sketch the graph of each line using the slope and y-intercept. Use graphing technology to check your graphs.

a) $y = 2x - 3$ $\qquad$ **b)** $y = -4x + 8$

c) $y = -x + 1$ $\qquad$ **d)** $y = \frac{5}{2}x - 4$

e) $y = -\frac{3}{4}x + 2$ $\qquad$ **f)** $y = 5$

4. Omar determines the slope and y-intercept of the line $3x - 2y - 8 = 0$. His work is shown.

$3x - 2y = 8$ Step 1

$-2y = -3x + 8$ Step 2

$y = \frac{3}{2}x + 4$ Step 3

The slope of the line is $\frac{3}{2}$ and the y-intercept is 4. Step 4

a) In which line did Omar first make an error?

b) Correct Omar's work.

5. Express each equation in slope-intercept form. Write the slope and y-intercept of each line.

a) $2x + y = 6$

b) $3x + y + 9 = 0$

c) $5x + 6y = 8$

d) $6x - y = 4$

e) $7x - y + 9 = 0$

f) $8x - 4y = 3$

6. Write the equation of each line in the form $y = mx + b$.

a) slope $= -3$, y-intercept $= 2$

b) slope $= \frac{5}{6}$, y-intercept $= -4$

c) slope $= -0.75$, y-intercept $= -5$

d) slope $= 1$, y-intercept $= -7$

e) slope $= -1$, y-intercept $= 0$

f) slope $= 0$, y-intercept $= \frac{1}{3}$

7. Use the following equations to answer each question. Justify your answers.

Equation A: $y = 3x - 5$
Equation B: $y = -4x + 1$
Equation C: $y = x + 2$
Equation D: $y = -\frac{1}{2}x$

a) Which lines slope *up* from left to right?

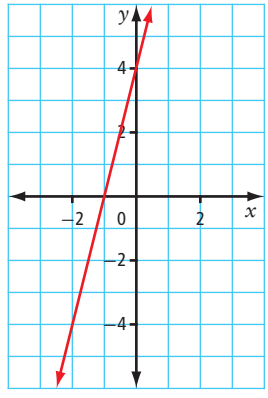
b) Which lines slope *down* from left to right?

c) Arrange the lines from greatest to least y-intercepts.

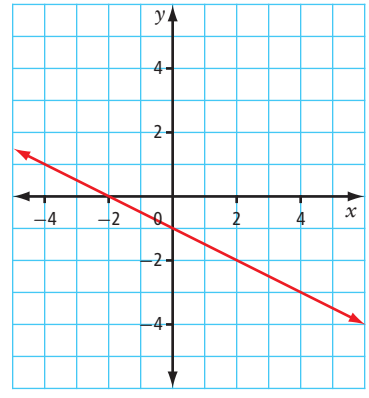
d) Which lines pass through the origin?

8. What are the slope and y-intercept of each line? Write the equation of each line in slope-intercept form.

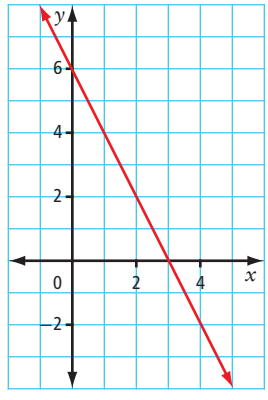
a)



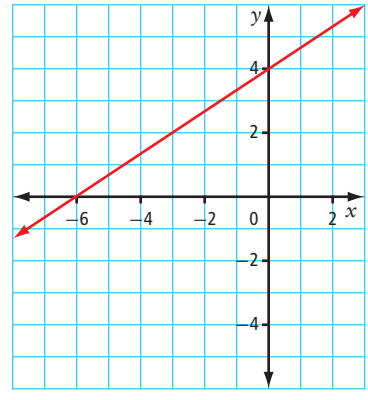
b)



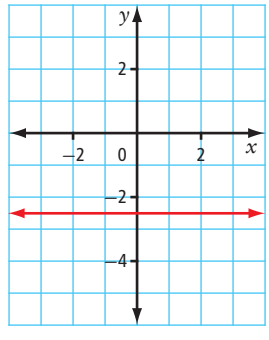
c)



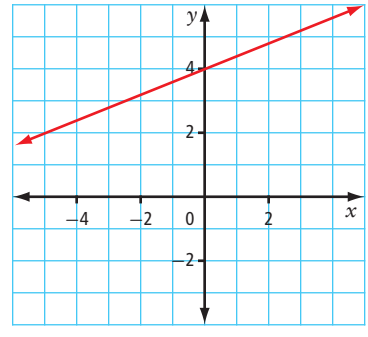
d)



e)



f)



9. Consider the equation $y = 3x + b$. What is each value of b if a graph of the line passes through each point?

a) $(4, 9)$ **b)** $(-4, 6)$

c) $(3, -2)$ **d)** $(-1, -8)$

10. For the equation $y = mx - 2$, what is each value of m if the line passes through each point?

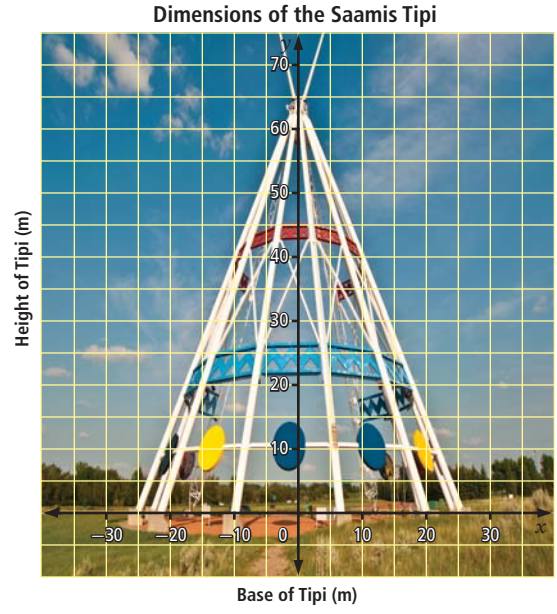
a) $(3, 1)$ **b)** $(-2, 8)$

c) $(4, -8)$ **d)** $(-6, -1)$

Did You Know?

The Saamis Tipi, a stylized structure, is located in Medicine Hat, AB. It was originally constructed for the 1988 Olympics as a tribute to First Nations peoples.

11. A photograph of the Saamis Tipi is shown on a metre grid. Write the equation of each line representing the extreme left post and the extreme right post. Express the equations in slope-intercept form.



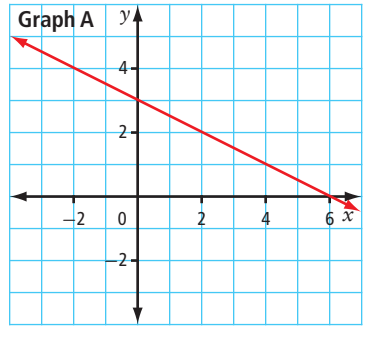
12. State the slope and y-intercept of each equation. Then, identify the graph that matches the equation. Use graphing technology to check your answers.

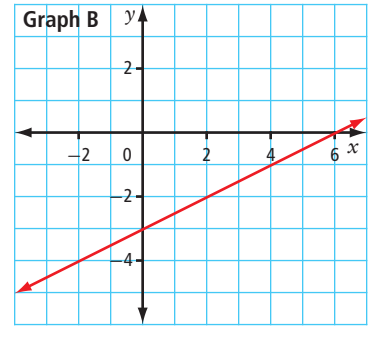
a) $y = -2x + 3$

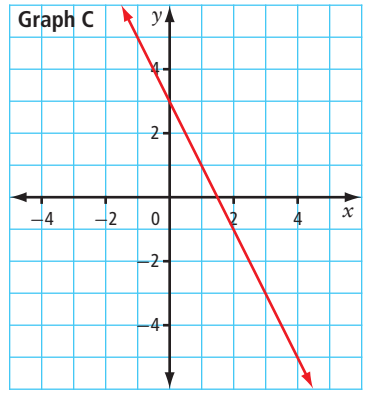
b) $y = 2x - 3$

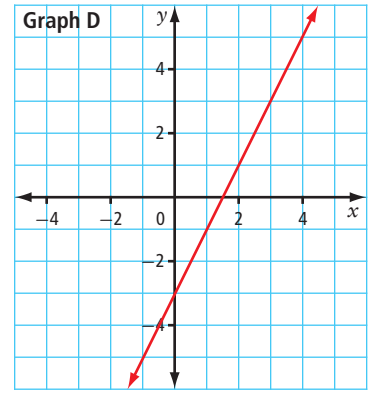
c) $y = \frac{1}{2}x - 3$

d) $y = -\frac{1}{2}x + 3$









13. Write an equation to represent each situation.

a) The cost, C, to take n students to the theatre is \$300 to rent a bus and \$6.25 per ticket.

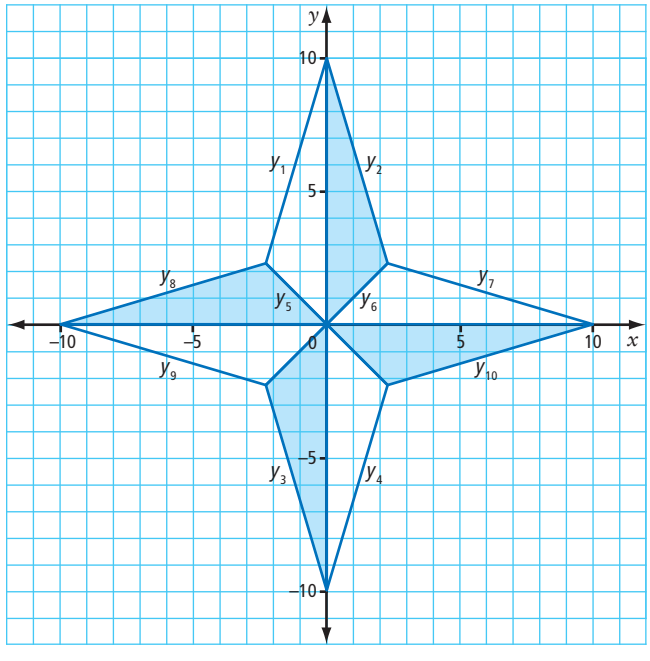
b) The taxi fee, T, is \$3.60 to start plus \$1.48 for each kilometre travelled, x.

c) A rewritable Blu-ray disc has 1024 MB of data stored on it. When new data is added to the disc, the total data, D, in megabytes, stored on the disc at time t seconds increases at a rate of 54 MB/s.

d) A water delivery truck is filling the water tank in Simeonie's house. The truck arrived with 2500 L of water. The number of litres of water, L, remaining in the truck at time t minutes decreases at a rate of 120 L/min.

Apply

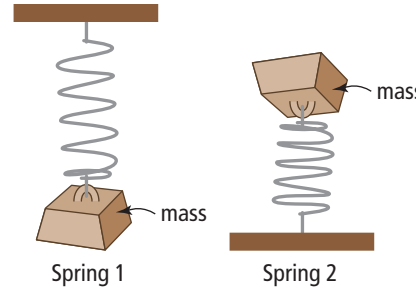
14. At the centre of the North Atlantic Treaty Organization (NATO) emblem is a compass that may be created using the ten line segments, labelled y_1 to y_{10}, in the figure. Work with a partner to determine the equation of each line segment. Express your equations in the form $y = mx + b$. For each equation, do not consider the boundaries of the line segment.



Did You Know?

Canada has been a member of NATO since NATO's formation in 1949. The main goal of NATO is to safeguard the freedom and security of its member countries.

15. A group of students tested how different masses changed the lengths of two different coil springs. The results of their experiments are summarized in the table.



Mass (g)	Spring 1 Length (cm)	Spring 2 Length (cm)
0	8	24
4	14	18
8	20	12
12	26	6

a) For each spring, write an equation to model how spring length, L, in centimetres, changes with mass, x, in grams. Express each equation in slope-intercept form.

b) What does a negative slope represent in the experiment?

Did You Know?

Many animals that roamed Earth at the time of the last glacial period are now extinct. These include the sabre-toothed cat, mammoth, mastodon, and giant beaver, bison, and bear. Other animals that roamed the area that is now known as western Canada during this time include the horse, camel, antelope, and ground sloth.

16. About 12 000 years ago during the last glacial period, giant bison roamed the plains of North America. Using fossil bones, paleontologists can estimate the size of these huge animals. The equation $y = 2.4x - 7.9$ approximates the relationship between an adult male bison's front limb length, y, in centimetres, and the length of its humerus bone, x, in centimetres.

Painting of giant bison by Ludo Beogaet

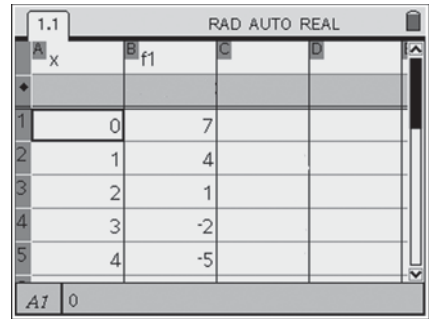
a) Sketch a graph of the equation $y = 2.4x - 7.9$.

b) Use graphing technology to check your graph. Then, estimate, to the nearest centimetre, the front limb length of each bison:

- an extinct giant bison with a fossil humerus bone length of 40.2 cm
- a modern North American bison with a humerus bone length of 32.6 cm

c) By what percent was the giant bison taller than the modern bison?

17. Marge uses graphing technology to graph a line. A table of values for the line is shown. What is the equation of the line? Express the equation in slope-intercept form.



18. **Unit Project** Exposure to cold weather can cause frostbite and hypothermia. Mountain climbers, sky divers, and other high-altitude enthusiasts must protect their skin because air temperature decreases as altitude increases. This rate of decrease in temperature is nearly constant, up to about 11 000 m.

An airplane taking off in the Yukon Territory recorded the following temperatures.

Altitude (m)	Temperature (°C)
0	12
4000	−13.6
8000	−39.2

a) Sketch a straight-line graph of the data.

b) What is the slope of the line expressed as a fraction in lowest terms? What does the slope represent?

c) What is the y-intercept? What does it represent?

d) Write an equation that describes the relationship between temperature, T, in degrees Celsius, and altitude, A, in metres.

e) Mount Logan is in Kluane National Park and Reserve, YT. At a height of 5959 m, Mount Logan is the highest mountain in Canada. It is the second highest mountain in North America. Predict the temperature a climber would experience at the peak of Mount Logan on the day that the airplane collected the data. Assume minimal wind.

f) Most people are at risk of frostbite within 10−20 min of exposure to temperatures below −20 °C. Predict the altitudes at which the temperature will be below −20 °C.

Did You Know?

The coldest recorded temperature in North America was −63 °C on February 3, 1947. It was recorded in the village of Snag, YT. Temperatures at higher altitudes can get even colder.

19. Write the equation of a line with the same slope as the line $x + 4y + 8 = 0$ and the same y-intercept as the line $2x - 3y + 12 = 0$.

Extend

20. When graphed, the equations $x + 4y = n$ and $5x - 2y = 10$ have the same y-intercept. What is the value of n?

21. A graph of $6x - ny = 8$ and $2x + 3y = 12$ shows two lines that have the same slope. What is the value of n?

22. A line has a y-intercept of 3 and forms an angle of 45° with the x-axis. Write the equation of this line in the form $y = mx + b$.

23. Consider the equation $2x + y = 200$. When graphed, how many points (x, y) on the line will have natural numbers for both the x-coordinate and y-coordinate?

Create Connections

24. Explain how to determine the slope and y-intercept of a line given each representation.

a) a table of (x, y) values

b) a graph of the line

c) an equation of the line in the form $y = mx + b$

25. Explain how you could use the slope and y-intercept of a line to

a) write the equation of the line

b) create a graph of the line

Materials

- graphing technology or grid paper and ruler

26. MINI LAB

Step 1 Use technology to graph each set of equations on the same display. Then, sketch a graph of each set.

a) $y_1 = x + 5$, $y_2 = -2x + 5$, $y_3 = 3x + 5$

b) $y_1 = x - 1$, $y_2 = 2x - 2$, $y_3 = -3x + 3$

c) $y_1 = -x + 2$, $y_2 = -x + 4$, $y_3 = -x + 6$

Step 2 Describe the similarities and differences between each set of lines. Lines that share at least one characteristic are called a *family of lines*. Why might each set be considered a family of lines?

Step 3 Write the equation of another line that belongs to each family. Use graphing technology to check your answers.

Step 4 Create your own family of lines. Write the equations of the lines in slope-intercept form. Share with a partner and describe each other's family of lines.