6.5 Slope

Focus on ...

- determining the slope of a line
- using slope to draw lines
- understanding slope as a rate of change
- solving problems involving slope

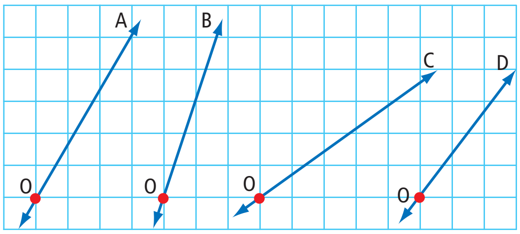
The national, provincial, and territorial parks of western and northern Canada feature some of the most beautiful backcountry in the world. To safely enjoy mountain adventures, specialized skills and knowledge, such as avalanche awareness, are essential. Though avalanches occur mostly in winter, they can happen at any time of the year. It is important to understand the many conditions that cause avalanches. The steepness, or slope, of a mountainside is one of them.

Materials

- grid paper
- plastic transparent ruler
- toothpick
- tape

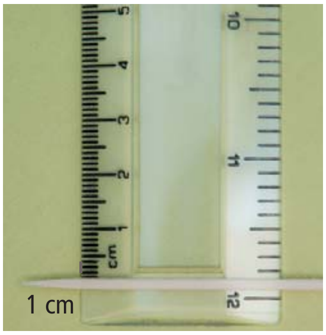
Investigate Slope

1. By observation, arrange the lines shown in order of steepness, from least steep to steepest. Explain your reasoning.

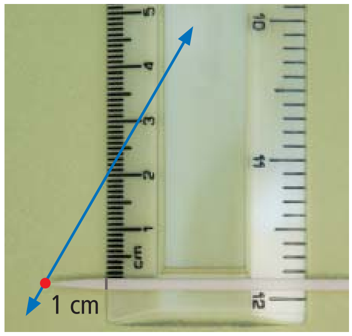


2. Convert a regular ruler into a "slope ruler" by taping a toothpick to the end of the ruler as follows:

- Make a pencil mark 1 cm from the end of the toothpick.
- Tape the toothpick so that its edge is aligned with the 0-cm mark of the ruler and the pencil mark is aligned with the edge of the ruler.



3. a) Use your slope ruler to measure and record the slope of each line. Place your ruler so that it is vertical and the end of the toothpick is on point O. Record the slope as the point where the line intersects the ruler.



b) How do the slope values that you measure relate to the steepness of each line?

4. On grid paper, plot a point and label it A. Measure 5 cm straight up (vertical) from point A and plot another point. Label this point B. Label the distance from A to B.

5. Measure 5 cm to the left (horizontal) of point A and 5 cm to the right (horizontal) of point A. Plot a point after each measurement. Label these points C and D. Label the distance from A to C.

6. a) Using a ruler, connect points C, B, and D. This triangle is a model mountain. Determine the ratio of the height at the centre of the mountain (A to B) to the horizontal distance from the centre to the base (A to C). That is, determine $\frac{AB}{AC}$.

b) Use your slope ruler to measure the slope of the mountain. How does the ratio compare to the slope given by the ruler?

7. a) Construct two other model mountains. Make one three times as high as the first one (AB = 15 cm) but with the same horizontal width (AC = 5 cm). Make the other the same height as the first one (AB = 5 cm) but with twice the horizontal width (AC = 10 cm).

b) Is each mountain steeper or less steep than the one in step 6?

c) Do you expect the slope value of each to be greater or less than the slope value in step 6?

d) Compare the ratio $\frac{AB}{AC}$ of each mountain to the slope value given by using the ruler.

8. **Reflect and Respond**

a) How do you draw a mountain with dimensions different from the model in step 6 but with the same slope? Draw this mountain. Check the slope by using your ruler.

b) A student uses a slope ruler and measures the slope of a model to be 8. If the model has a height of 32 cm at the centre, what is the distance from the centre to the base of the model?

The horizontal distance for this slope ruler is 1 cm. When the slope ruler measures a slope of 8, this represents the ratio of $\frac{8}{1}$.

c) Can a slope ruler give a measurement of $\frac{1}{4}$? If so, explain how.

d) Can a slope ruler show a negative value? If so, what does that value look like?

e) Suppose the horizontal distance for a slope ruler was 2 cm instead of 1 cm. What slope would a height reading of 8 cm represent on this slope ruler?

Link the Ideas

The **slope** of a line or line segment tells you how steep the line is. The sign of the slope value indicates the direction of the line.

slope
- the ratio of the vertical change, or rise, to the horizontal change, or run, of a line or line segment
- not expressed with units

$$\text{slope} = \frac{\text{vertical change}}{\text{horizontal change}}$$

$$\text{or } m = \frac{\text{rise}}{\text{run}} \text{ or } m = \frac{\Delta y}{\Delta x}$$

m is the variable used for slope and Δ is a symbol used to indicate change. The expression Δ*y* is read as "delta *y*."

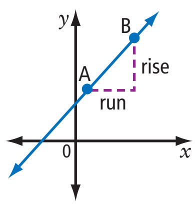
A line or line segment that rises from left to right has a positive slope.

Move from point A to point B:

$$\frac{\text{positive vertical change}}{\text{positive horizontal change}} \text{ results in a positive slope}$$

Move from point B to point A:

$$\frac{\text{negative vertical change}}{\text{negative horizontal change}} \text{ results in a positive slope}$$



slope is positive

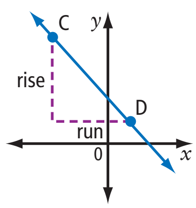
A line or line segment that falls from left to right has a negative slope.

Move from point C to point D:

$$\frac{\text{negative vertical change}}{\text{positive horizontal change}} \text{ results in a negative slope}$$

Move from point D to point C:

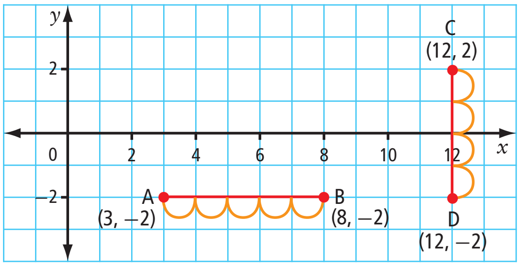
$$\frac{\text{positive vertical change}}{\text{negative horizontal change}} \text{ results in a negative slope}$$



slope is negative

Determining the Slope Using Points on a Line

When the distance from one point to another is along a horizontal line or a vertical line, you can find the distance by simply counting the spaces on the grid or by using subtraction. For example,

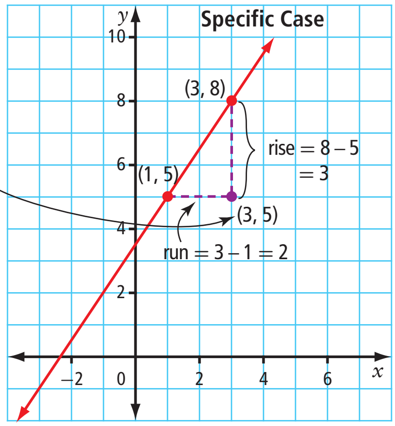


By counting, the distance AB is 5 units.
By subtraction, the distance AB is $8 - 3 = 5$ units.

By counting, the distance CD is 4 units.
By subtraction, the distance CD is $2 - (-2) = 4$ units.

Applying this idea, you can develop a formula to find the slope of any line.

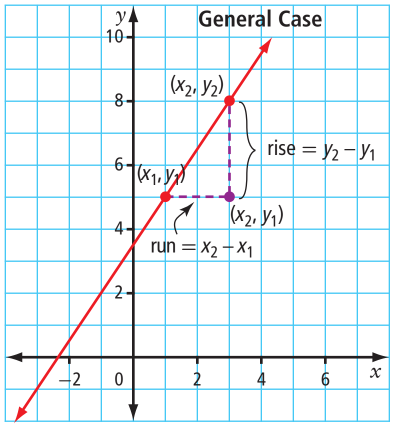
Notice how the coordinates of this point are related to the coordinates of the two points on the line.



Slope formula

$m = \frac{\text{rise}}{\text{run}}$

$m = \frac{3}{2}$



Slope formula

$m = \frac{y_2 - y_1}{x_2 - x_1}, x_2 \neq x_1$

Applying the slope formula to line AB above shows that the slope of a horizontal line is 0.

$m = \frac{\text{rise}}{\text{run}}$

$m = \frac{0}{8}$

$m = 0$

Applying the slope formula to line CD above shows that the slope of a vertical line is undefined.

$m = \frac{\text{rise}}{\text{run}}$

$m = \frac{6}{0}$

$m = \text{undefined}$

Example 1 Classify the Slope of a Line

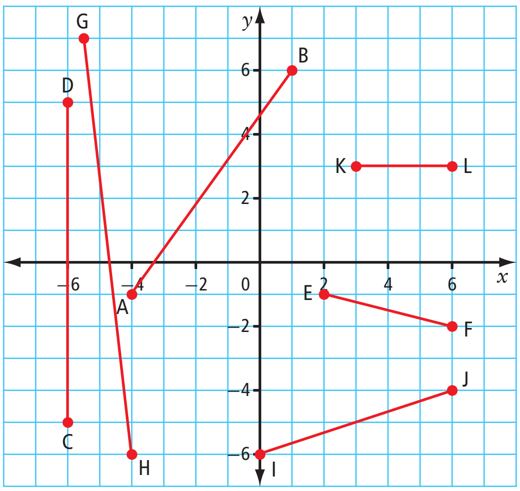
The North Shore in Vancouver is popular for hiking and biking. Bridges and stunt structures on trails are complex and often extremely challenging. They have a huge variety of slopes. Classify each slope marked on the photographs as either positive or negative.

Solution

Lines and line segments that rise from left to right have positive slopes. Therefore, line segment AB has a positive slope. Lines and line segments that fall from left to right have negative slopes. Therefore, line segments CD and EF have negative slopes.

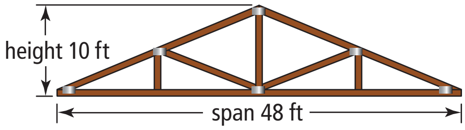
Your Turn

Classify the slope of each line segment as positive, negative, or neither.



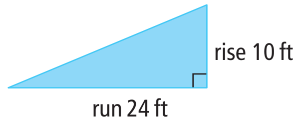
Example 2 Determine the Value of a Slope

When discussing a roof truss, carpenters refer to the *span* instead of the *width*. They talk about the *pitch* rather than the *slope*. Determine the pitch of the roof supported by the truss shown. Explain the meaning of your answer.



Solution

The pitch of the roof is its slope.



$$m = \frac{\text{rise}}{\text{run}}$$

$$m = \frac{10}{24}$$

How is the run determined?

$$m = \frac{5}{12}$$

The pitch of the roof is $\frac{5}{12}$. This means that the roof rises 5 units for every 12 units of horizontal distance.

Your Turn

Suppose that the roof truss in Example 2 has a height of 1 m and a span of 8 m. Determine the pitch and explain your answer.

Example 3 Determine the Slope of a Line Segment

What is the slope, *m*, of each line segment with the given end points?

a) S(−3, 6) and T(5, 2)
b) H(4, 3) and K(4, 8)
c) M(−9, −7) and N(−1, −7)

Solution

Method 1: Use a Graph

Plot the points on grid paper. Count the rise and run.

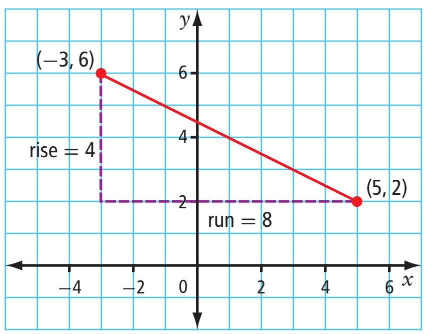
a) Plot the points (−3, 6) and (5, 2).

$$m = \frac{\text{rise}}{\text{run}}$$

$$m = -\frac{4}{8}$$

$$m = -\frac{1}{2}$$

Recall that a line that falls from left to right has a negative slope.



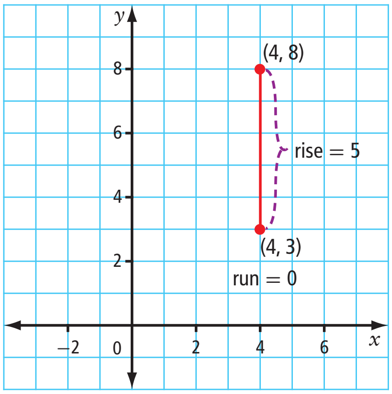
b) Plot the points (4, 3) and (4, 8).

$m = \frac{\text{rise}}{\text{run}}$

$m = \frac{5}{0}$

m is undefined

Division by zero is not defined in the real number system.

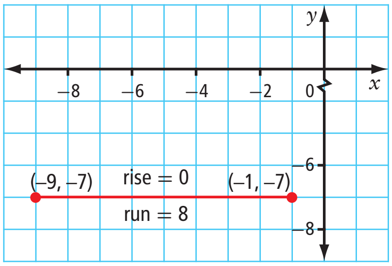


c) Plot the points (−9, −7) and (−1, −7).

$m = \frac{\text{rise}}{\text{run}}$

$m = \frac{0}{8}$

$m = 0$



Method 2: Use the Slope Formula

Label the points and substitute into the formula.

a) S(−3, 6) T(5, 2) or T(5, 2) S(−3, 6)

$m = \frac{y_2 - y_1}{x_2 - x_1}$ $\qquad$ $m = \frac{y_2 - y_1}{x_2 - x_1}$

$m = \frac{2 - 6}{5 - (-3)}$ $\qquad$ $m = \frac{6 - 2}{-3 - 5}$

$m = \frac{-4}{8}$ $\qquad$ $m = \frac{4}{-8}$

$m = -\frac{1}{2}$ $\qquad$ $m = -\frac{1}{2}$

b) H(4, 3) K(4, 8)

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$m = \frac{8 - 3}{4 - 4}$

$m = \frac{5}{0}$

m is undefined

It does not matter which point is selected as (x_1, y_1); the value of the slope is unaffected.

c) N(−1, −7) M(−9, −7)

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$m = \frac{-7 - (-7)}{-9 - (-1)}$

$m = \frac{0}{-8}$

$m = 0$

Your Turn

a) Use a graph to determine the slope of the line segment with endpoints P(−5, 6) and Q(1, 10).

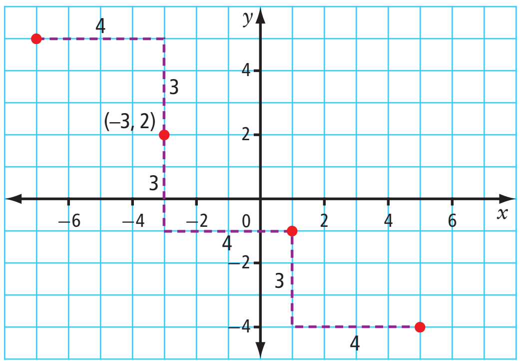
b) Use the slope formula to determine the slope of the line segment with endpoints W(2, −2) and X(−5, 5).

Example 4 Use Slope to Graph a Line

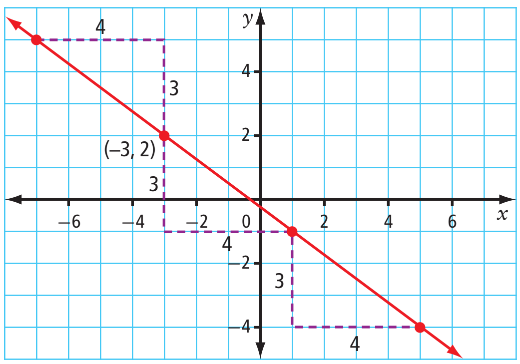
The point (−3, 2) is on a line that has a slope of $-\frac{3}{4}$. List three other points on the line. Graph the line.

Solution

The slope of the line gives the rise and run from one point to another. Plot the point (−3, 2). From this point, use the slope to locate other points on the line. Since the slope is negative, move down 3 units and right 4 units, or move up 3 units and left 4 units.



Three other points on the line are (−7, 5), (1, −1), and (5, −4). Now draw the line through the points.



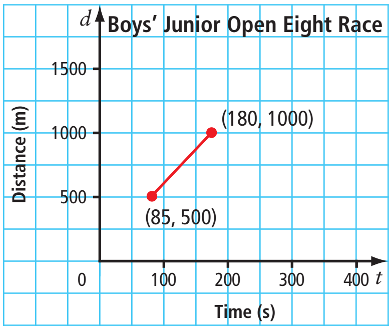
Move down 6 units and right 8 units from the point (−3, 2). What do you notice? Explain.

Your Turn

The point (−6, 1) is on a line that has a slope of $\frac{1}{3}$. List three other points on the line and graph the line.

Example 5 Slope as a Rate of Change

The Brentwood Regatta in Mill Bay, BC, is the largest junior rowing regatta hosted by a single school in North America. The races are all 1500 m in length. The graph shows the approximate times at the 500-m mark and the 1000-m mark for one of the boys' races. Determine the average rate of change for this portion of the race.



Solution

The slope of the line segment gives the ratio of the change in distance to the change in time. For this portion of the race,

$$\text{Rate of change} = \frac{\Delta d}{\Delta t}$$

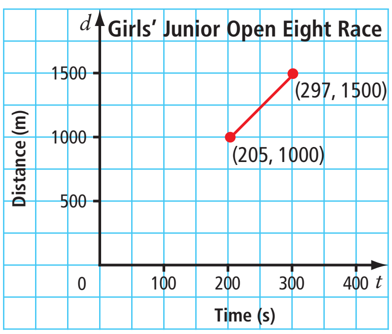
$$\text{Rate of change} = \frac{(1000 - 500)}{(180 - 85)}$$
$$= \frac{500}{95}$$

To help you interpret the meaning of the rate of change, look at the units that are used. This rate of change represents the rowers' average speed.

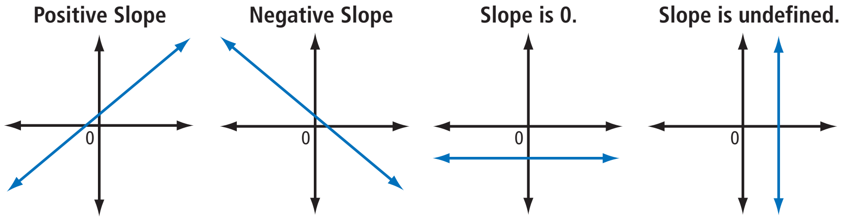
The average rate of change is approximately 5.26 m/s.

Your Turn

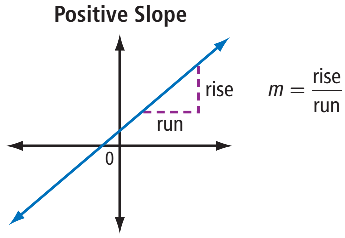
The graph shows the approximate times at the 1000-m mark and at the 1500-m mark for a rowing crew of the girls' junior open eight race at the Brentwood Regatta. Determine the average rate of change for this portion of the race.



Key Ideas

- 

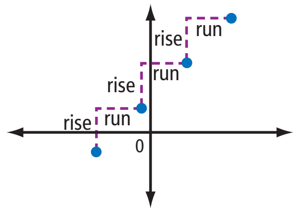
- The slope of a line is the ratio of the rise to the run.



- The slope of a line can be determined using two points on the line, (x_1, y_1) and (x_2, y_2).

  $m = \dfrac{y_2 - y_1}{x_2 - x_1}, x_2 \neq x_1$

- If you know one point on the line, you can use the slope to find other points on the line.



- The slope gives the average rate of change.

Time (s)	Distance (m)
1	4
2	7
3	10
4	13
5	16
6	19
7	22

Rate of change $= \dfrac{\Delta d}{\Delta t}$

Rate of change $= \dfrac{3}{1}$

The average rate of change is 3 m/s.

Time (s)	Distance (m)
1	4
3	10
5	16
7	22

Rate of change $= \dfrac{\Delta d}{\Delta t}$

Rate of change $= \dfrac{6}{2}$

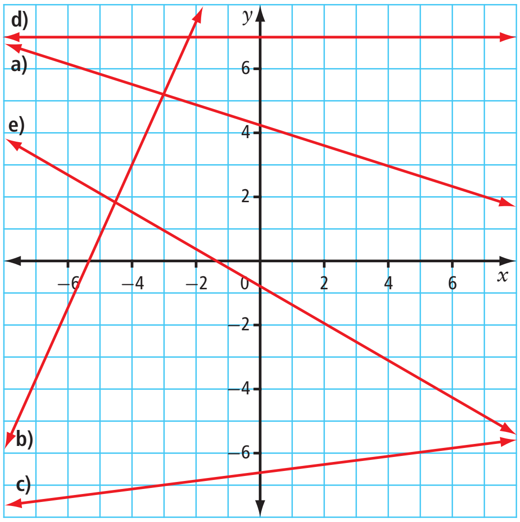
Rate of change $= \dfrac{3}{1}$

The average rate of change is 3 m/s.

Check Your Understanding

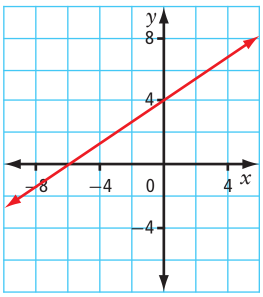
Practise

1. For each line, identify the slope as positive, negative, or zero.

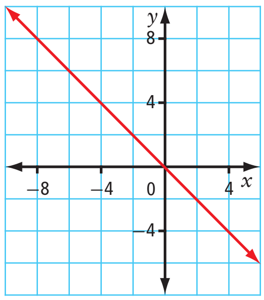


2. Determine the slope of each line.

a)



b)



3. Use the slope formula to determine the slope of the line passing through each pair of points.

a) (2, 4), (9, 8)

b) (1, 12), (6, 12)

c) (−2, −5), (1, −7)

d) (3, 6), (−3, −12)

e) (−9, 16), (−9, 25)

f) (3.9, 10.6), (10.3, 13.8)

4. Graph each line, given a point on the line and its slope.

a) (2, 3), $m = -\frac{2}{3}$

b) (−4, −4), $m = \frac{3}{5}$

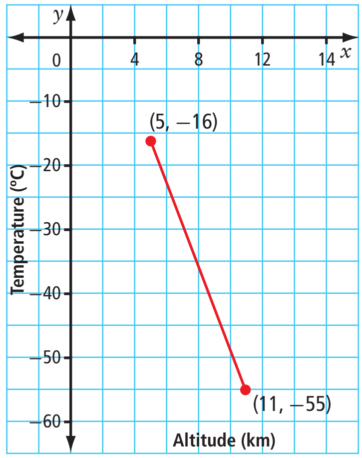
c) (5, −3), $m = -4$

d) origin, $m = \frac{1}{4}$

e) (−1, 6), $m = 0$

f) (4, 0), $m =$ undefined

5. The graph shows the air temperature at different altitudes above Earth's surface. Determine the average rate of change.



Apply

6. Time and height values (seconds, metres) are given for the Free Fall Slide in Kenosee, SK. Determine your average rate of change if you go down this slide.

7. **a)** Create a graph showing the melting of a 75-cm-high snow bank in spring. Plot the height, in centimetres, of the snow bank on the vertical axis and time, in days, on the horizontal axis. Draw a segment with a slope of −3, with one endpoint at (0, 75) and the other endpoint along the horizontal axis.

   b) What does each point on the graph represent?

   c) What does the endpoint along the horizontal axis represent?

   d) Explain the meaning of the slope in this situation.

8. Marjorie is having a wheelchair ramp built at the front entrance of her house.

   a) The rise to Marjorie's front door is 18 in. What is the shortest run, x, allowed for the ramp if the building code in her town sets a maximum slope of $\frac{1}{12}$?

   b) How long is the ramp?

   c) How long would the ramp be if Marjorie decides to have a gentler slope of $\frac{1}{16}$?

9. This sign on the Trans-Canada Highway indicates that a steep hill is ahead.

a) Written as a ratio, what is the slope of the hill, as indicated by the sign?

b) Describe this slope in words.

10. The Penny Ice Cap glacier in Auyuittuq National Park on Baffin Island, NU is melting. In 2009, some areas of the glacier were about 1000 ft thick. It is estimated that if the glacier continues to melt at its current rate, the ice cap could be 967 ft thick by 2020. What is the estimated rate of change in thickness?

Did You Know?

Auyuittuq National Park was established in 1976. To honour the Penny Ice Cap, the people of Pangnirtung gave the park its name, Auyuittuq. This means The Land That Never Melts.

11. In 1800, the wood bison population in North America was estimated at 168 000. The population declined to only about 250 animals in 1893. That year, Wood Buffalo National Park was established on the Alberta/Northwest Territories border. In 2006, there were about 5600 bison in the park.

a) What was the average rate of change in the bison population from 1800 to 1893? Describe the meaning of this rate.

b) What was the average rate of change in the bison population from 1893 to 2006? Describe the meaning of this rate.

12. The mountain pine beetle is infesting many forests in British Columbia and Alberta. In 2004, about 1000 infested trees were counted in Alberta. In 2007, the number of infested trees in the province was about 2.8 million.

a) Determine the average rate of change per year.

b) What does this rate of change represent?

c) What assumptions did you make? Predict the number of infested trees in Alberta in 2012.

Did You Know?

The roof of the speed skating oval for the Vancouver 2010 Winter Olympics is made almost entirely of wood. The wood is from pine trees that had been infested by the mountain pine beetle.

13. Since the speed of light is faster than the speed of sound, you see lightning before you hear the sound of the thunderclap. If a thunderstorm is 1100 m away, the sound of thunder is heard in 3.2 s. If the storm is 4950 m away, the sound reaches you in 14.5 s.

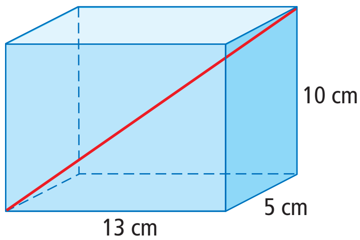
a) Determine the average rate of change, to the nearest metre per second.

b) What does this rate of change represent?

c) If you hear thunder 30 s after you see lightning, how far away is the storm?

Extend

14. What is the slope from the bottom front corner of the box to the top back corner of the box shown?



15. A metal cube has a side length of 5 cm. The cube is heated, causing it to expand uniformly to a side length of 5.01 cm.

a) Determine the volume of the cube before and after heating.

b) Determine the average rate of change of the cube's volume with respect to its side length.

16. The points that a line passes through are given as algebraic expressions, $(-4x, 7x^2)$ and $(8x, 15x^2)$. Determine a simplified algebraic expression for the slope of the line.

Create Connections

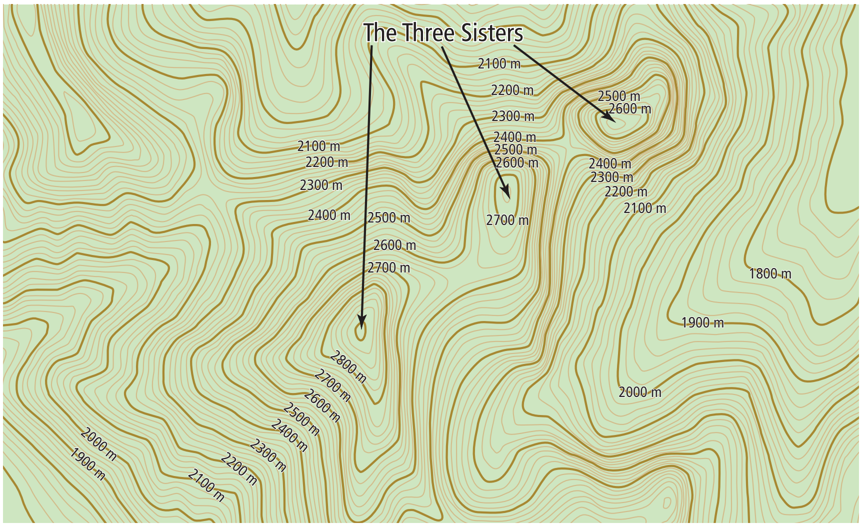
17. Explain why the slope of a line is *constant*. Use the terms *rise* and *run* in your explanation.

18. Matthew measured the slope of a ramp to be $\frac{1}{16}$. He then used trigonometry to determine the angle that the ramp made with the ground.

a) Describe how he did this.

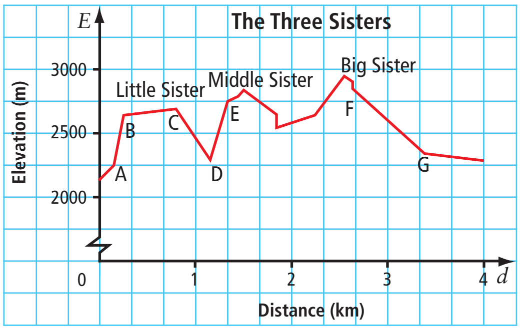
b) Determine the angle.

19. **MINI LAB** Topographic maps show hills and valleys using contour lines. Contour lines connect points of equal elevation. Contour lines are usually labelled with the elevation above sea level, as shown in this sample map.



Step 1 The map shows contour lines and selected elevations for The Three Sisters mountains in southern Alberta. If the change in elevation between two adjacent lines is 20 m, what is the approximate height of each peak?

Step 2 The diagram is a simplified cross profile, or side elevation, of the Three Sisters. Compare the diagram and the map. Which peak on the map represents Big Sister?



Step 3 The slope of a mountain will vary from the bottom to the top. Explain how to estimate the average slope of Middle Sister.

Step 4 Suppose the greatest risk for an avalanche occurs when the slope is between 0.5 and 1.7. From the diagram, determine the approximate slope of the following sections of the Three Sisters: AB, BC, CD, DE, and FG. Which section(s) pose the greatest avalanche risk?

Step 5 The risk of an avalanche is reduced if the slope of the mountainside is less than 0.5 or greater than 1.7. Explain why this statement is true.

WWW Web Link

To learn more about avalanche awareness and safety, go to www.mhrmath10.ca and follow the links.