6.3 Domain and Range

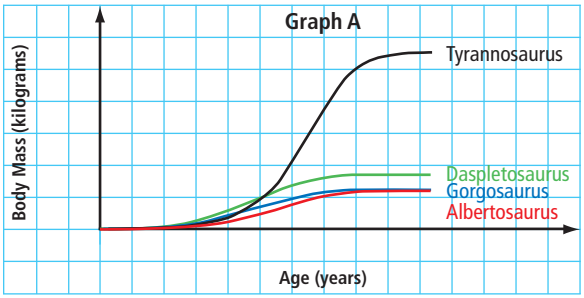
Scientists study our world to piece together the story of our past and make predictions about our future. Their writings are often accompanied by graphs to illustrate the data they are presenting. The more detail given with the graphs, the easier it is for the reader to interpret the information. One important aspect of a graph is the span of possible values for each quantity being compared.

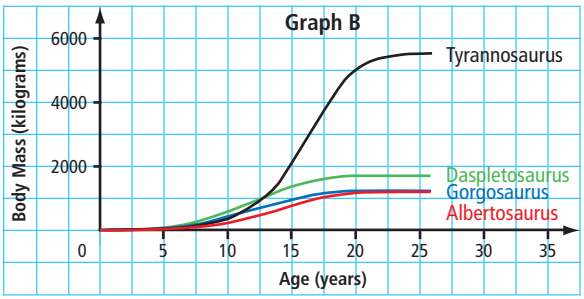
Focus on ...

- understanding the meaning of domain and range
- expressing domain and range in a variety of ways

Investigate Appropriate Values for the Dependent and Independent Variables

1. By studying changes in several specimens of tyrannosaurs, paleontologists can estimate the lifespan and growth rates of the species. The graphs show the relationship between body mass and age for four types of tyrannosaur.





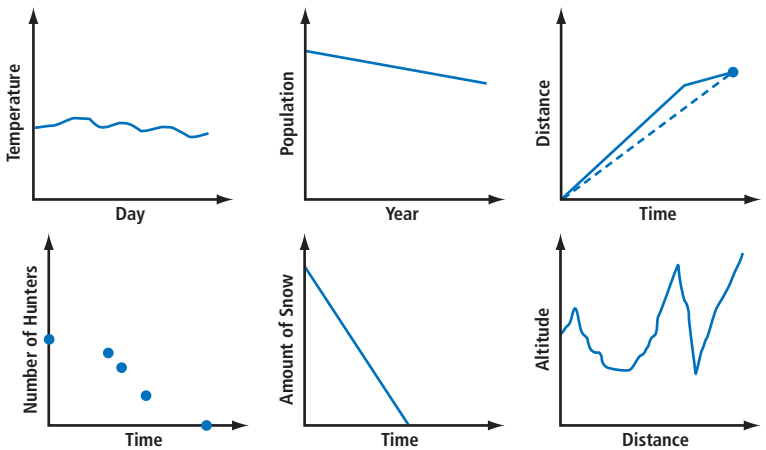
a) If you are given only Graph A, what conclusions can you make about the relationship between each tyrannosaur's body mass and its age?

b) What additional information can you tell if you are given Graph B instead?

2. Read the following story inspired by the Dene legend for the creation of the seasons.

> The first people of Earth had to endure **winter for twelve months** of the year. Most of the land was covered by massive, moving layers of ice and deep snow. No vegetation survived in the harsh gripping cold. All water was frozen in a land of endless cold.
>
> One day, when the first people were hunting, they came upon a bear with a sack around his neck. The curious hunters asked what was in the sack, and the bear growled that it was filled with the abundance of summer's warmth and light. The hunters coveted the sack, but the bear would not part with it. They decided to lure the bear to a great feast of moose and **caribou**, fill him with food, and, when he slept, steal the sack. The bear readily accepted their invitation for a feast, but arrived without the sack around his neck. The bear ate his fill and fell asleep. The frustrated chief ordered **four of the village's skilled hunters to follow the bear home** and steal the sack by any means. Peering inside the bear's large cave next morning, the hunters spotted the sack on the cave floor with two black bears guarding it. The courageous hunters sprang into the cave to demand the sack. **A fierce fight killed three of the hunters and mortally wounded the fourth**, but before he died, he grabbed the sack and unleashed the abundance of warmth and light. Instantly, the air became warm and the sky filled with bright sunlight. **The snow melted** into rivers and lakes, and the **hills and valleys** became covered with trees, flowers, and bushes. Strange birds flew in great numbers and built nests, and streams filled with fish. Every year since that time, summer has come to the Dene.
>
> *Author unknown*

The general graphs are related to the **bold text** in the story. In small groups, discuss how each graph depicts the information from the story.



3. **a)** Work with a partner. Choose any four of the graphs. Agree upon appropriate values for the dependent and independent variables. Clearly describe these possible values using words, lists, or number lines.

   b) Compare your results with those of another group. Share your reasoning with them.

4. **Reflect and Respond**

   a) Why did you not choose the two graphs that you did not use?

   b) Did the other group choose any graphs different from yours? If so, do you agree with their choice of possible values for each axis? Explain.

   c) How does adding values to each axis aid in the understanding of the graphs?

Link the Ideas

domain
- the set of all possible values for the independent variable in a relation

range
- the set of all possible values for the dependent variable as the independent variable takes on all possible values of the domain

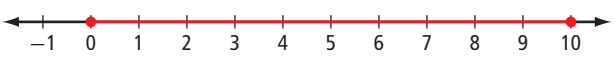
When comparing two quantities, the words **domain** and **range** are used to describe the values that are appropriate.

In a set of ordered pairs, values for the domain are the first element of each pair. Values for the range are the second element. On a graph, values of the domain are plotted along the horizontal axis. Values of the range are plotted along the vertical axis.

There are a variety of ways to express the domain and range of a relation.

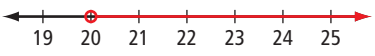
- **Words** can be used to describe the values that are allowed. For example, the domain is the set of all real numbers between 0 and 10, inclusive. The range is the set of all real numbers greater than 20.

- **Number lines** give a picture of the values that are allowed. For example, this number line represents all numbers between 0 and 10, inclusive.



The solid circles at 0 and 10 indicate that these points are included.

This number line represents all numbers greater than 20.



The open circle at 20 indicates that the value 20 is not included. The arrow pointing to the right indicates that there is no upper limit.

This number line represents the discrete list of numbers −2, 0, 4, 8, and 10.

- **A list** is a useful way to give the domain and range for discrete data when there are not many numbers in the set.
  For the relation (0, 0), (1, 5), (3, 7), (5, 7), the domain is {0, 1, 3, 5} and the range is {0, 5, 7}.

- **Set notation** is a formal mathematical way to give the values of the domain and range.

Set Notation	What It Means
The domain: $\{x \mid x \leq 10, x \in R\}$	{ } is the type of brackets used for a set. $\in$ means "is an element of". $\mid$ means "such that". The statement is read as follows: x is an element of the real numbers such that x is less than or equal to 10.
The range: $\{y \mid y > 20, y \in R\}$	The statement is read as follows: y is an element of the real numbers such that y is greater than 20.

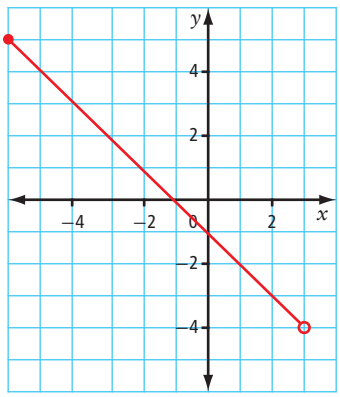
Recall that the symbols used for the number sets are
R for real numbers
Q for rational numbers
$\overline{Q}$ for irrational numbers
I for integers
N for natural numbers
W for whole numbers

- **Interval notation** uses different brackets to indicate an interval.
  This style of bracket,], is used if the end number is included.
  This style of bracket,), is used if the end number is not included.
  The infinity symbol, ∞, is used if there is no end point.
  A domain of all numbers between 0 and 10, inclusive, would be given as [0, 10].
  A range of all numbers greater than 20 would be given as $(20, \infty)$.

Example 1 Determine the Domain and Range From a Graph

For each graph, give the domain and range. Use words, a number line, interval notation, and set notation.

a)



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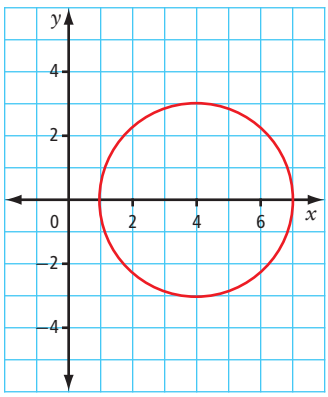


Solution

a) From looking at the graph, you can see that the smallest value for x is -6. The largest value for x is up to, but not including, 3. The smallest value for y is down to, but not including, -4. The largest value for y is 5.

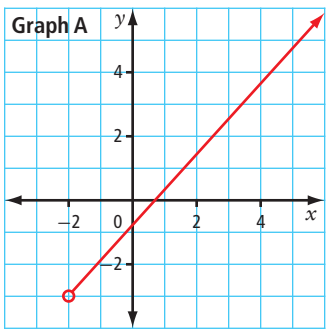
Domain	Range
Words All real numbers between -6 and 3, including -6 but not including 3	*Words* All real numbers between -4 and 5, not including -4 but including 5
Number Line  	*Number Line* 
Interval Notation $[-6, 3)$	*Interval Notation* $(-4, 5]$
Set Notation $\{x \mid -6 \leq x < 3, x \in \mathrm{R}\}$	*Set Notation* $\{y \mid -4 < y \leq 5, y \in \mathrm{R}\}$

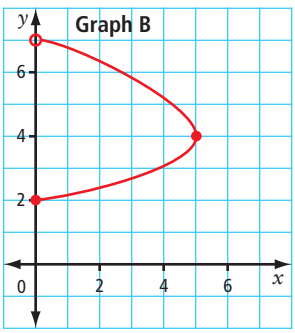
b) From looking at the graph, you can see that the smallest value for x is 1. The largest value for x is 7. The smallest value for y is -3. The largest value for y is 3.

Domain	Range
Words All real numbers between 1 and 7, inclusive	*Words* All real numbers between -3 and 3, inclusive
Number Line  	*Number Line* 
Interval Notation $[1, 7]$	*Interval Notation* $[-3, 3]$
Set Notation $\{x \mid 1 \leq x \leq 7, x \in \mathrm{R}\}$	*Set Notation* $\{y \mid -3 \leq y \leq 3, y \in \mathrm{R}\}$

Your Turn

For each graph, give the domain and range using words, a number line, interval notation, and set notation.





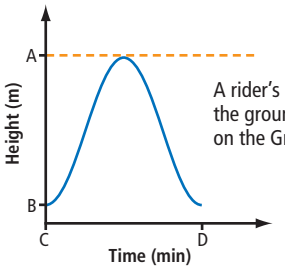
Example 2 Domain and Range for a Situation

The Great Wheel is being built in Beijing in the People's Republic of China. When finished, it will be the largest Ferris wheel in the world. The wheel will have a diameter of 193 m and will reach a maximum height of 208 m.

The graph shows a rider's height relative to the ground for a 20-min ride through one rotation.

a) What are the values of points A, B, C, and D, and what do they represent?

b) What are the domain and the range of the graph?



A rider's height above the ground for one ride on the Great Wheel.

Solution

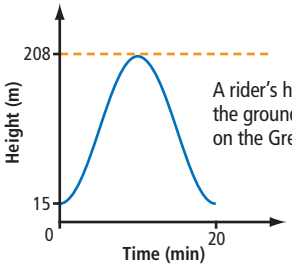
a) Point A is the highest point on the graph, so it must represent the highest point on the ride. Its value is 208 m.

Point B is the lowest point on the graph. It represents the location where a rider boards the Ferris wheel. Since the diameter of the wheel is 193 m, this point is 193 m below point A. By subtraction, the value of B is 15 m.

Point C is the origin, (0, 0). It represents ground level on the vertical axis and the starting time on the horizontal axis.

Why does the graph not begin at (0, 0)?

Point D represents the time it takes to complete one revolution. Its value therefore is 20 min.



A rider's height above the ground for one ride on the Great Wheel.

Why is the graph in quadrant I?

b) The domain and range can be described in several ways.

Words:

Domain: All times are between 0 min and 20 min, inclusive.
Range: A rider's height above the ground is between 15 m and 208 m, inclusive.

Number Line:

Domain: Ride time, in minutes

Range: Height above the ground, in metres

Interval Notation:

Domain: Ride time, in minutes: $[0, 20]$
Range: Height above the ground, in metres: $[15, 208]$

Set Notation:

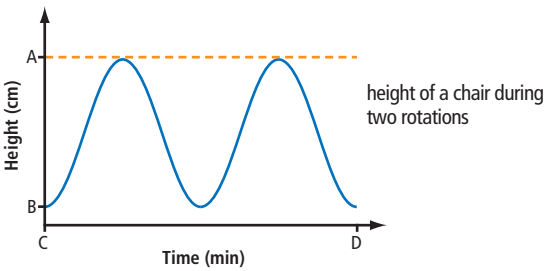
Domain: Ride time, in minutes: $\{t \mid 0 \leq t \leq 20, t \in \mathrm{R}\}$
Range: Height above the ground, in metres: $\{h \mid 15 \leq h \leq 208, h \in \mathrm{R}\}$

Your Turn

A motorized model Ferris wheel has a radius of 22 cm. The support structure keeps the bottom of the wheel 3 cm above the base. It takes 10 s to complete one revolution. The graph shows the height of one of the chairs during two rotations of the wheel, starting at the lowest point.

a) What are the values of A, B, C, and D? What do they represent?

b) What are the domain and range of the graph? Express each in words, as a number line, in interval notation, and in set notation.



WWW Web Link

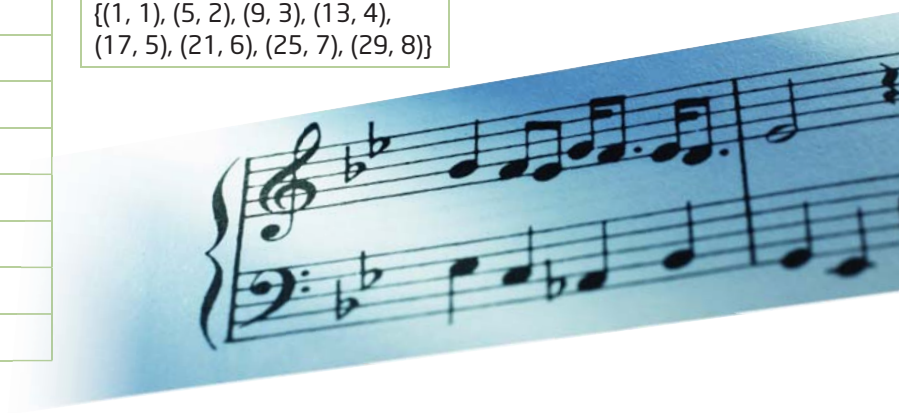
To view a graph of a rider's height above the ground when on a Ferris wheel, go to www.mhrmath10.ca and follow the links.

Example 3 Domain and Range for Discrete Data

Caitlin is marking time for some music by clapping on the first beat of every bar. The table of values and the set of ordered pairs show the relationship between the total number of beats and her total number of claps. Give the domain and range of the relation using words and a list.

Number of Beats	Number of Claps
1	1
5	2
9	3
13	4
17	5
21	6
25	7
29	8

Ordered Pairs (Beats, Claps)
{(1, 1), (5, 2), (9, 3), (13, 4), (17, 5), (21, 6), (25, 7), (29, 8)}



Solution

Words:

Domain: The total number of beats is given by the numbers 1, 5, 9, 13, 17, 21, 25, and 29.

Range: The total number of claps is given by the whole numbers between 1 and 8, inclusive.

List:

Domain: {1, 5, 9, 13, 17, 21, 25, 29}

Range: {1, 2, 3, 4, 5, 6, 7, 8}

Your Turn

Data for a relation are recorded in the table of values.
Give the domain and range using set notation and lists.

a	*b*
−3	5
−2	6
−1	7
0	8
1	9
2	10

Example 4 Use Technology to Graph a Relation With a Restricted Domain

A variety of corn plant grows at an average rate of 4.5 cm per day from the start of the third week of growth to the end of the sixth week. The plant's growth can be modelled using the formula $h = 4.5a + 25$, where h is the height of the plant, in centimetres, and a is the age of the plant, in days. Use a graphing calculator to create a graph of a cornstalk's height from the beginning of week 3 to the end of week 6.

Solution

To get a view of the plant's height for the required time frame, you need to restrict the window settings of the graphing calculator displaying the relation $h = 4.5a + 25$.

Which variable is the independent variable? How do you know?

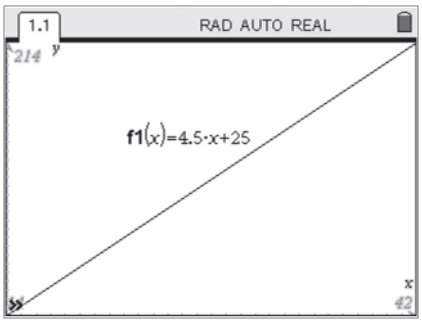
In the domain you want only the values for the age of the plant during the period from week 3 to week 6, inclusive. At the beginning of week 3, the plant is 14 days old. At the end of week 6, the plant is 42 days old. Therefore, the domain is [14, 42].

The range is the set of all possible plant heights during this growth period.
If $a = 14$, then $h = 4.5(14) + 25$ or $h = 88$.
If $a = 42$, then $h = 4.5(42) + 25$ or $h = 214$.
Therefore, the range is [88, 214].

The model is valid only for these ages.

Graph the relation on a graphing calculator.

- Enter the equation of the relation.
- Set the values for the window settings.
- Graph the relation.

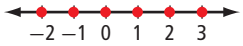


Your Turn

The same species of corn grows at an average rate of 5 cm per day from the start of week 7 until the end of week 9. The plant's growth in this period is modelled using the formula $h = 5a + 214$, where h is the height of the plant, in centimetres, and a is the age of the plant, in days. Use a graphing calculator to show a graph of the plant's height for these three weeks.

Key Ideas

- The domain of a relation is the set of all numbers for which the independent variable is defined.
- The domain of a relation may also be described as:
  - the set of first coordinates in a set of ordered pairs
  - the possible values in the first column of a table of values
  - the possible values on the horizontal axis of a graph
- The range of a relation is the set of all numbers for which the dependent variable is defined.
- The range of a relation may also be described as:
  - the set of second coordinates in a set of ordered pairs
  - the possible values in the second column of a table of values
  - the possible values on the vertical axis of a graph
- The domain and range can be expressed in different ways.

Words	All integers equal to or greater than -2 and less than or equal to 3
Number Line	
Interval Notation	$[-2, 3]$
Set Notation	$\{n \mid -2 \leq n \leq 3, n \in \mathrm{I}\}$
A List	$\{-2, -1, 0, 1, 2, 3\}$

Check Your Understanding

Practise

1. Describe the set of numbers indicated by each number line. Use words, interval notation, and set notation.

a) 

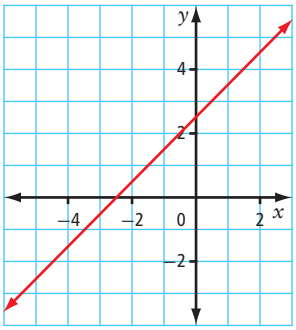
b) 

c) 

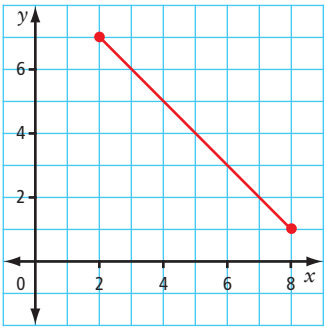
d) 

2. Give the domain and range of each graph. Use words, a number line, interval notation, and set notation.

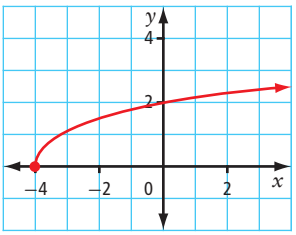
a)



b)



c)



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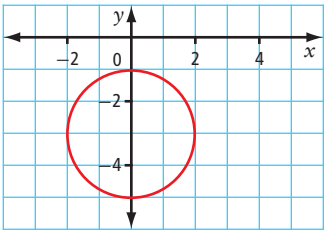

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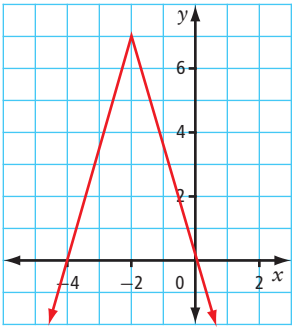

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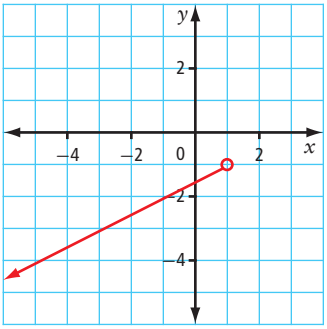
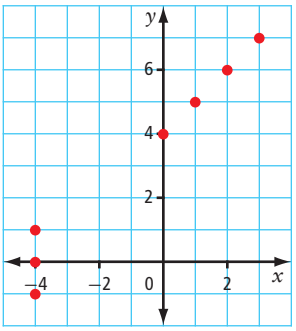


3. Write the domain and the range of each relation as lists.

a)



b)

s	*v*
−4	5
−2	5
0	7
2	7
4	9
6	9

c) (50, 10), (100, 20), (150, 30), (200, 40)

4. A relation is given by the formula $k = 2.8m - 3.5$.

a) If the domain of the relation is [0, 25], what is the range?

b) Graph the relation on a graphing calculator. Record the window values you chose. Sketch the relation.

Apply

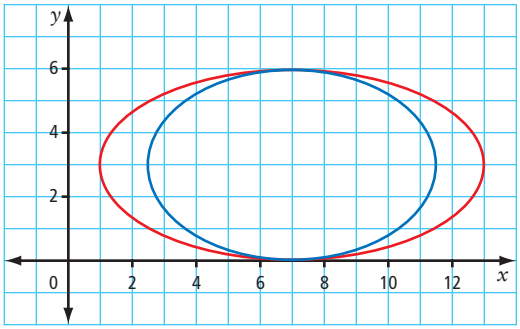
5. The table gives the average annual high temperature for a number of western and northern Canadian cities.

City	Average Annual High Temperature (°C)
Winnipeg	8.3
Regina	9.1
Edmonton	8.5
Calgary	10.5
Vancouver	13.7
Victoria	14.1
Whitehorse	4.5
Yellowknife	−0.2

a) Give the domain and the range for the relation.

b) Graph the relation.

6. A company offers two models of above-ground oval swimming pools. The ovals in the graph are drawn using the dimensions, in metres, of the pools.



a) Using interval notation, what are the domain and range of the blue oval?

b) Using set notation, what are the domain and range of the red oval?

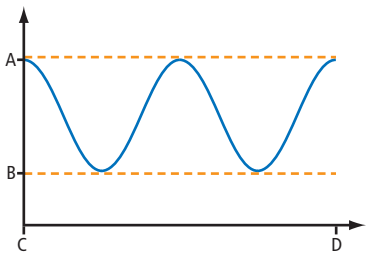
c) What are the actual dimensions of each pool?

7. An electric car can travel 193 km before its battery needs to be recharged.

a) Create a graph to show the distance travelled over time if the car is driven at an average speed of 60 km/h.

b) Describe the domain and the range of your graph in two ways.

8. The graph shows the changes in tide levels for Bella Coola on the central coast of British Columbia for a 24-h period starting at 12:00 a.m. The greatest water depth, at high tide, is 15.9 ft. The least depth, at low tide, is 4.5 ft.



a) Sketch a copy of the graph. Label each axis with appropriate dependent and independent variables.

b) What are the values of A, B, C, and D? What do they represent?

c) What are the domain and range of the graph? Express these in words, as a number line, in interval notation, and in set notation.

9. A hot-air balloon is flying at an altitude of 1236 m. It begins to descend at a rate of 10 m per minute.

a) How long does it take the balloon to reach the ground?

b) Assign variables to represent each quantity in the relation. Identify the independent variable and the dependent variable. Graph the balloon's progress from the moment it begins its descent until it reaches the ground.

c) Does the graph continue in both directions? Explain.

d) What do the domain and range represent in this context?

e) Give the domain and the range using all forms appropriate for this situation.

Extend

10. The domain of a linear relation is given by $\{x \mid -2 \leq x \leq m, x \in \mathrm{I}\}$. The range, in order, is given by $\{2, 6, 10, k, 18\}$. Use a graphing approach to determine the smallest possible values of m and k.

11. James can store 8 GB of music on his phone. If a 3-min digital song requires 3.5 MB of storage space, what are the domain and range for the relationship between memory used and number of songs that are each 3 min in length?

Create Connections

12. Using everyday language, how do you describe domain and range to someone not familiar with the concepts?

13. For a topic that interests you, such as sports, music, or entertainment, give an example of a relation that has a restricted domain and range. Why do these restrictions exist?