6.2 Linear Relations

Focus on ...

- determining if a relation is linear
- representing linear relations in a variety of ways
- explaining why data points should or should not be connected
- identifying the dependent and independent variables in a relation

Knowing the relationship between two quantities can be very useful. For example, it benefits a business to know how the number of units it sells is related to the product's price. It is important for a civil engineer to know the relationship between structural design and strength. Medical doctors are interested in the relationship between a patient's body mass index and heart health.

People have long studied relationships. The famous artist and scientist Leonardo da Vinci (1452–1519) studied the proportions in the human body. Why do you think this information is important to an artist?

Leonardo da Vinci

Investigate Relationships in the Human Body

Materials

- measuring tape
- ruler
- grid paper or graphing technology

1. Work with a partner.
   a) Measure and record each other's height, in centimetres.
   b) Measure and record the length, in centimetres, of each other's foot.
2. How many of your feet does it take to equal your height? How about your partner's?
3. a) As a class, share your results and determine the mean. Using this mean, predict a relationship between a person's height and foot length. Show your answer to the nearest whole foot.
   b) Write this relationship as an equation. This is your model **relation**.
4. Estimate the shortest length and longest length of the feet of high-school students. Use these estimates to complete a table of values. Use increments of 1 cm for foot length. Determine the corresponding height using your equation from step 3b).

relation

- an association between two quantities
- can be presented in words, as an equation, as ordered pairs, as a table of values, or as a graph

5. Look at the height values in your completed table. Are they appropriate to represent the heights of high-school students? If not, make adjustments to your foot-length estimates.

6. **a)** Plot your results from step 4 to see a graph of this relationship. Record foot length on the horizontal axis and height on the vertical axis.
   b) Should you draw a line through the points on the graph? Why or why not?
   c) Is the resulting graph a straight line or a curve? Explain your answer.
   d) Determine the difference between each height value in your table. What do you notice?

7. **Reflect and Respond**
   a) What are different ways to present a relationship?
   b) If a relation is presented as a table of values, how do you determine if the relation is linear without creating a graph? Is the relationship between a person's height and shoe size linear? Explain your answer.
   c) Should a graph of this relation have the data points connected? Explain your answer.

Link the Ideas

Relations

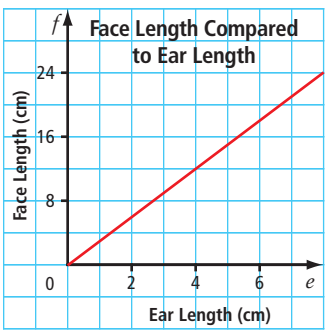
A relation can be presented in a variety of ways. For example,

Words	Equation	Ordered Pairs
Three times the length of your ear, e, is equal to the length of your face, f, (from chin to hairline).	$f = 3e$	(4, 12), (4.5, 13.5), (5, 15), (5.5, 16.5), (6, 18), (6.5, 19.5)

Table of Values

Ear Length, e (cm)	Face Length, f (cm)
4	12
4.5	13.5
5	15
5.5	16.5
6	18
6.5	19.5

Graph



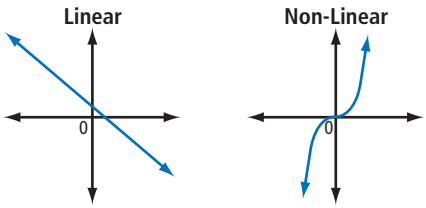
Linear and Non-Linear Relations

There are a number of ways to determine whether a relation is a **linear relation** or a **non-linear relation**.

linear relation
- a relation that forms a straight line when the data are plotted on a graph

non-linear relation
- a relation that does not form a straight line when the data are plotted on a graph

Linear relations have graphs that are straight lines.



You can determine whether a relation is linear or non-linear from a table of values. In linear relations, values of y increase or decrease by a constant amount as values of x increase or decrease by a constant amount. Horizontal and vertical lines are exceptions.

Linear Relation

	x	y	
	2	8	
+1	3	11	+3
+1	4	14	+3
+1	5	17	+3

Non-Linear Relation

	x	y	
	2	8	
+1	3	27	+19
+1	4	64	+37
+1	5	125	+61

When a linear relation is written as an equation, it will contain one or two variables and its degree will be 1.

For an expression to have degree 1, what must be the maximum sum of the exponents of the variables for any term?

Linear Relations	Non-Linear Relations
$x = 7$	$2x + y^2 = 6$
$3m + 2n = -12$	$h = k^3$
$y = -\frac{2}{3}x + 5$	$xy = 3$

Discrete or Continuous Data

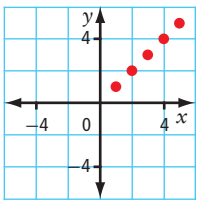
A graph of **discrete data** can only show points because the values in between them have no meaning. A graph of **continuous data** is a solid line or curve.

discrete data
- data values on a graph that are not connected

continuous data
- data values on a graph that are connected

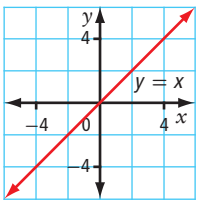
For example, a relation is defined by the set of ordered pairs {(1, 1), (2, 2), (3, 3), (4, 4), (5, 5)}. There are only five data points in the relation. These are discrete data. The graph has five unconnected points.

Discrete Data



For the relation defined by the equation $y = x$, there are an infinite number of possible ordered pairs. The points (1, 1), (2, 2), (3, 3), (4, 4), and (5, 5) satisfy this relation. So do many other points such as $\left(\frac{3}{2}, \frac{3}{2}\right)$ and (−3.6, −3.6). These represent continuous data. On a graph, you show an infinite set with an unbroken, or continuous, line.

Continuous Data



independent variable
- the variable for which values are selected

dependent variable
- the variable whose values depend on those of the independent variable

Independent and Dependent Variables

In a relation with two variables, one is the **independent variable** and the other is the **dependent variable**.

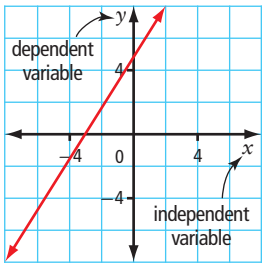
When a relation is expressed as a table of values, the values of the independent variable are listed in the first column. The values of the dependent variable are listed in the second column.

x	$y = 3x + 5$
−1	2
0	5
1	8
2	11

Choices for independent variable

Corresponding values of dependent variable

When a relation is expressed as a graph, the values of the dependent variable are plotted along the vertical axis. The values of the independent variable are plotted along the horizontal axis.



Example 1 Describe a Relation in a Variety of Ways

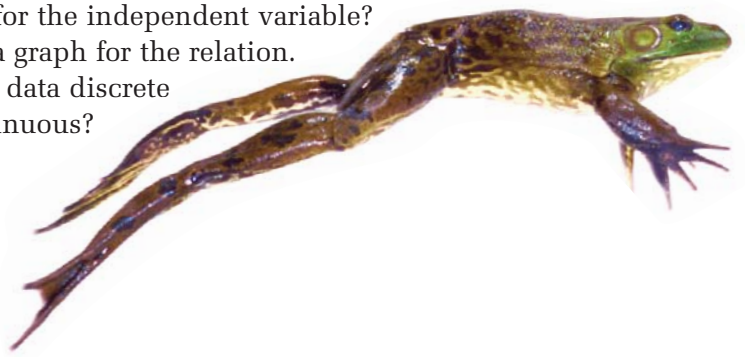
The Canadian National Frog Jumping Championship is part of Les Folies Grenouilles. This annual festival is in St-Pierre-Jolys, MB. The first champion, a frog named Georges, jumped a distance of just over 2 m in a single leap. Assume that Georges could maintain a distance of 2 m on every jump and that the total distance travelled from the start is measured after every jump. Consider the relationship between the number of jumps Georges takes and the total distance the frog travels.

a) Identify the relationship as linear or a non-linear. Explain how you know.

b) Create a variable to represent each quantity in the relation. Which is the dependent variable? Which is the independent variable?

c) Create a table of values for this relation. What are appropriate values for the independent variable?

d) Create a graph for the relation. Are the data discrete or continuous?



Solution

a) Since the distance that Georges covers on each leap is the same, the relation is linear.

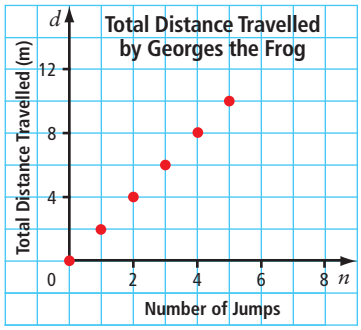
b) The total distance travelled depends on how many jumps the frog takes. Let n represent the independent variable, the number of jumps. Let d represent the dependent variable, the distance travelled.

c) Choose a realistic number of consecutive jumps that Georges might make. For example, the frog could make five jumps.

Why can the values of n only be whole numbers?

n	d
0	0
1	2
2	4
3	6
4	8
5	10

d) Display the independent variable, n, on the horizontal axis and the dependent variable, d, on the vertical axis. The data are discrete because there are only six possible values in the relation. Georges does not take partial jumps, so values for n such as 1.5 or 2.8 cannot be used.



Your Turn

Another popular event at *Les Folies Grenouilles* is the fireworks display. Assume that the event organizers send off 20 firework shells each minute.

a) Is the relationship between the total number of fireworks and the duration of the event linear or non-linear? Explain how you know.

b) Assign a variable to represent each quantity in the relation. Which variable is the dependent variable? Which is the independent variable?

c) Create a table of values for this relation. What are appropriate values for the independent variable?

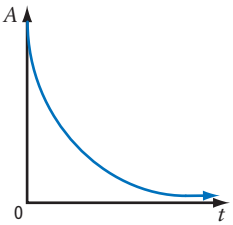
d) Create a graph for the relation. Are the data discrete or continuous?

Example 2 Determine Whether a Relation Is Linear or Non-linear

Consider each relation. Determine whether the relation is linear. Explain why or why not.

a) the relation described by {…, (−9, −10), (−7, −5), (−5, 0), (−3, 5), (−1, 10), …}

b) The graph shows the relationship between the amount, A, of a radioactive isotope present and the age of a rock sample over time, t, in years.

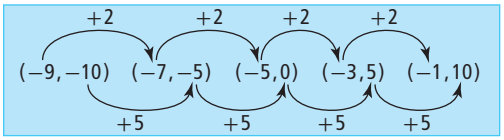


c) the relation described by the equation $m - 17 = 0.8n$

Solution

a) Method 1: Compare Changes in the Independent and Dependent Variables

Check to see if the independent variable increases or decreases at a constant rate and if, at the same time, the dependent variable increases or decreases at a constant rate.



The relation is linear. With each increase of 2 in the independent variable, the dependent variable increases by 5.

Method 2: Use a Table of Values to Compare Changes in Each Variable

	x	y	
+2	−9	−10	+5
+2	−7	−5	+5
+2	−5	0	+5
+2	−3	5	+5
	−1	10	

The relation is linear. Values of x (the independent variable) increase each time by 2. Values of y (the dependent variable) increase each time by 5.

b) The relation is not linear. The graph is not a straight line.

c) The degree is 1. The relation is linear.

How do you know that the degree is 1?

Your Turn

Determine whether each relation is linear. Explain why or why not.

a) the relationship between the cost to rent a dance hall and the number of people attending the dance, if the hall charges $200 plus $5 for each person who attends

b) the relation described by the equation $x^2 + y^2 = 25$

c) the relation described by the set of ordered pairs {(10, 12), (15, 4), (20, −4), (25, −12), (30, −20)}

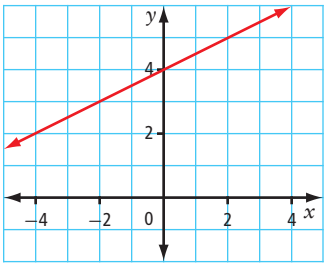
Example 3 Match Representations of a Linear Relation

Match each linear relation with possible representations in the selections that are given. Justify your choices.

a) The pressure, P, that a scuba diver experiences under water increases at a constant rate relative to the diver's depth, d, below the surface.

b) $y = \frac{1}{2}x + 4$

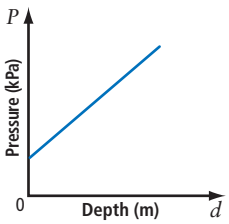
A



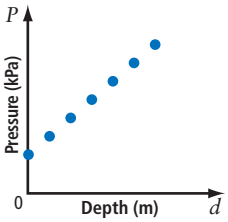
B

x	y
0	4
1	8
2	12
3	16

C



D

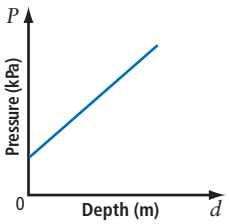


E One number is half another number increased by four.

F (0, 101), (25, 176), (50, 251), (75, 326), (100, 401), (125, 476)

Solution

a) The pressure, P, that a scuba diver experiences under water increases at a constant rate relative to the diver's depth, d, below the surface. The graph in choice D is a possible representation.



Increases in pressure are constant as depth changes. Therefore, the relation is linear. Also, since the values for the independent variable are not restricted to whole numbers of metres, the data are continuous.

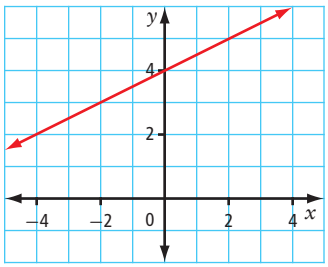
Did You Know?

The kilopascal is an SI unit for measuring pressure. Its abbreviation is kPa. This unit represents a force of 1000 N per square metre. The pascal is named after the French mathematician Blaise Pascal. In the 1600s Pascal experimented with barometers, which are instruments used to measure air pressure.

Blaise Pascal

b) $y = \frac{1}{2}x + 4$

The graph in choice A shows this relation.
The representation in choice E also expresses this relation in words.



The equation represents linear data and continuous data.

The points on this graph satisfy the relation.

For example, (−3, 2.5), (−2, 3), and (0, 4) are all solutions to the equation.

Your Turn

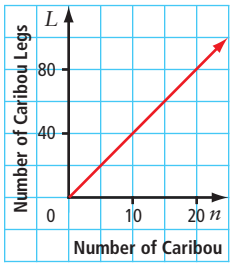
There is a linear relationship between the number of caribou, n, in a herd and the number of caribou legs, L. Which representations model this relation?

A $L = 4n$

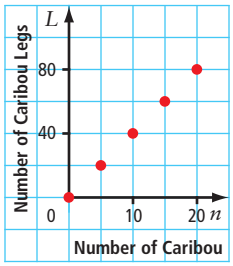
B (0, 0), (3, 12), (8, 32), (15, 60), (50, 200)

C $L = n + 4$

D



E

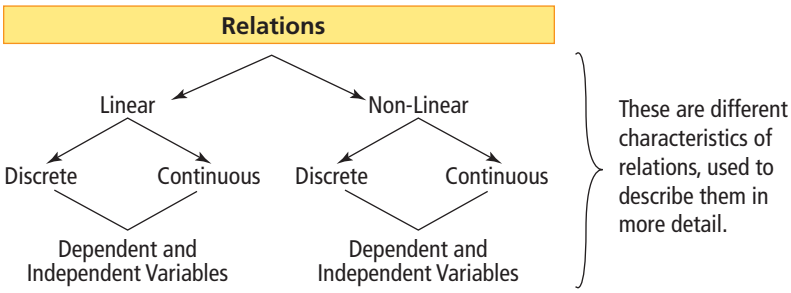


F

n	L
3	6
6	12
9	18
12	24

Key Ideas

- Relations can be represented in a variety of ways. You can use words, equations, tables of values, ordered pairs, or graphs.



Check Your Understanding

Practise

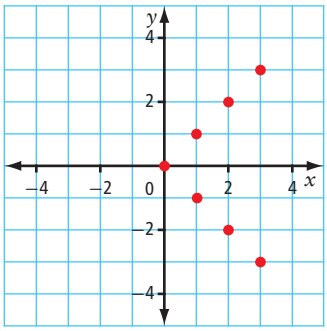
1. Convert each relation from its current representation to the one suggested.

a)

m	*n*
−2	5
−1	6
0	7
1	8
2	9
3	10
4	22

to ordered pairs

b)



to a table of values

c) $P = 2d + 5$ to a graph

d) (1, 1), (2, 2), (3, 3), (4, 4), (5, 5) to words, if the independent variable represents the number of children and the dependent variable represents the number of oranges eaten

2. Determine whether each relation is linear or non-linear. Explain your decision.

a) $C = 2\pi r$

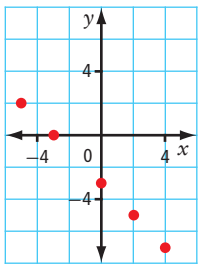
b) $A = s^2$

c) $y = 5x - 3$

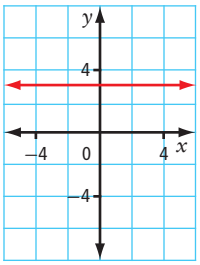
d) (0, 0), (1, 1), (4, 2), (9, 3), (16, 4)

e) (5, 10), (10, 20), (15, 30), (20, 40), (25, 50)

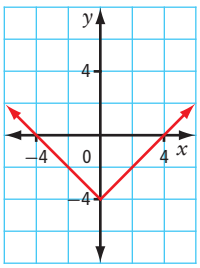
f)



g)



h)



3. For each relation, state the dependent variable and the independent variable.

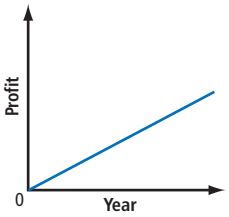
a) $A = \pi r^2$

b) $V = 7t + 2$

c)

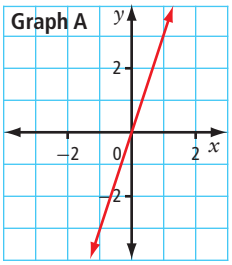
n	A
0	1
1	3
2	9
3	27
4	81

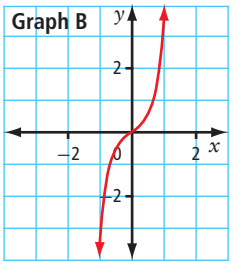
d)

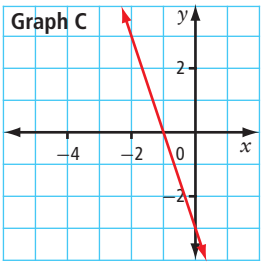


e) Five eggs, e, per chicken, c, are laid each week.

4. Consider the relation described by the equation $y = x^3 + 2x$ and the three graphs shown. Without performing any calculations, predict which graph matches this relation. Check your prediction against a partner's. Discuss your answers.







Apply

Did You Know?

A bushel is a unit used for measuring the volume of grain, fruit, vegetables, or other dry things. One bushel is equal to about 0.036 m^3.

5. More than half of all the wheat grown in western Canada is grown in Saskatchewan. One year, Saskatchewan farmers received \$6 per bushel for wheat. Consider the relationship between the total amount of money each farmer received and the number of bushels the farmer sold.

a) Is this a linear or non-linear relationship? Explain how you know.

b) Assign a variable to represent each quantity in the relation. Which variable is the dependent variable? Which is the independent variable? Explain how you know.

c) Assume that Saskatchewan wheat farms each produce up to 50 000 bushels of wheat. Create a table of values for the relationship.

d) Are the data discrete or continuous? Explain how you know.

e) Graph the relationship.

6. Give an example of a relation with each characteristic listed.

a) a table of values that shows a non-linear relationship

b) a graph that shows a non-linear discrete relationship

c) an equation that shows a linear relation with independent variable m and dependent variable n

7. The refraction of light causes an object lying under water to appear closer to the surface than it actually is. The relationship between how deep a coin appears to be in a fountain and the coin's actual depth is given by the equation $D = 0.75d$, where D is the apparent depth and d is the actual depth.

a) Is this relationship linear or non-linear? Explain.

b) Identify the dependent variable and the independent variable for this relation.

c) If the water is 2 m deep, how deep does the coin appear to be?

d) Can you use the formula to determine the apparent depth of the coin if the water is 2.8 m deep? Does this indicate that the relation is continuous or discrete? Explain.

8. A killer whale is swimming at a speed of 6 km/h. Consider the relationship between the total distance, in kilometres, travelled by the whale and time, in hours.

a) Assign variables to represent each quantity in the relation. Identify the dependent variable and the independent variable.

b) Assume that the whale swims for 5 h without stopping. Create a set of ordered pairs for the relation.

c) Is the relation continuous or discrete? Explain.

d) Graph the relation.

e) Is the relation linear or non-linear? Explain.

Did You Know?

Killer whales have long played a role in the legends and beliefs of coastal First Nations cultures. The Haida, for example, used the killer whale as a symbol for family. They did this because the whales stayed in families, travelling in large pods.

Killer Whale (Sgáan) by Don Yeomans, Haida artist

9. An action plan is in place to clean up a lake that has high levels of mercury. Tests show 9 mg of mercury per 1000 L of water. It is estimated that mercury levels can be reduced each year by 0.8 mg per 1000 L of water.

a) Create a table of values to represent the relationship between the amount, A, of mercury present and the number of years, t, since the clean-up plan started. A is the dependent variable and t is the independent variable in the relation.

b) According to this relation, how long will it take to rid the lake of mercury?

10. The Pacific Coast is the most earthquake-prone region in Canada, while Saskatchewan and Manitoba have the least probability of earthquakes.

Seismometer measuring seismic waves.

a) A magnitude scale describes the relative size of earthquakes. An earthquake with a magnitude value that is one higher than another earthquake is about 10 times more intense. For example, an earthquake measuring 7 is about 10 times as great as one with a magnitude of 6. Is the relationship between an earthquake's magnitude and its size linear or non-linear? Explain.

b) From northern Vancouver Island to the Queen Charlotte Islands, the oceanic Pacific plate is sliding to the northwest at about 6 cm per year. Starting with this year as 0, create a set of ordered pairs to show the relationship between the number of years from now and the amount of plate sliding for the next 8 years.

c) Graph the ordered pairs. Should the points be connected? Should you continue the pattern in both directions? Explain your answers.

Did You Know?

A seismometer is an instrument used to measure seismic waves. Information about the waves is then used to determine the magnitude of an earthquake.

Extend

11. Does each table of values represent a linear or a non-linear relation? Explain.

a)

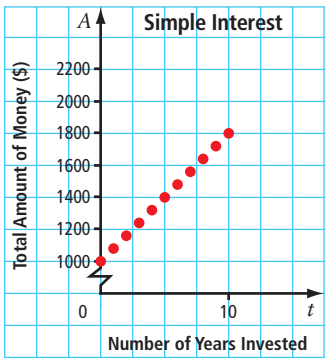
x	y
1	k
3	$2k$
5	$3k$
7	$4k$

b)

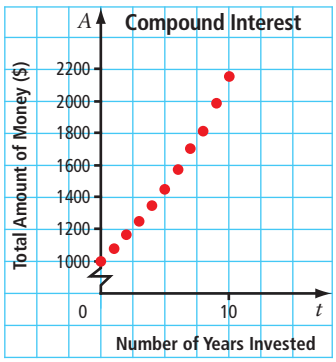
x	y
m	0
$m + 1$	$3n$
$m + 2$	$6n$
$m + 3$	$9n$

12. The graphs represent different investments based on investing $1000 today. The first graph shows simple interest of 8% per year. The second graph shows compound interest at 8% per year.

Graph A



Graph B



a) Which graph is linear? Which is non-linear? Explain.

b) In one interest calculation, you receive a set percent of your original investment each year. In the other interest calculation, you receive a set percent of the amount that you have in your account. Identify which graph represents each of these scenarios. Explain your reasoning.

Create Connections

13. Which method do you prefer to use when representing a relation? Explain.

14. Think about a relationship between two things that you are interested in learning more about. Describe two ways you could collect the information.