

Date: _____

4.6 Factoring Special Trinomials

Ex. Factor:

$$\begin{array}{l}
 4x^2 - 9 \\
 4x^2 + 0x - 9 \\
 \underline{4x^2 + 6x - 6x - 9} \\
 2x(2x+3) - 3(2x+3) \\
 (2x+3)(2x-3)
 \end{array}
 \quad
 \begin{array}{r}
 \otimes -36 \\
 \oplus 0 \\
 \hline
 6, -6
 \end{array}
 \quad
 \begin{array}{l}
 a^2 - 25b^2 \\
 a^2 + 0ab - 25b^2 \\
 \hline
 (a-5b)(a+5b)
 \end{array}
 \quad
 \begin{array}{r}
 \otimes -25 \\
 \oplus 0 \\
 \hline
 -5, 5
 \end{array}$$

← conjugates →

Difference of Squares: a binomial expression involving the subtraction of perfect squares

ex. $4q^6 - 169$, $64z^{12} - 16b^4$

Pattern can be applied to both multiplication of conjugates and factoring of difference of squares.

Ex. Multiply:

$$\begin{array}{l}
 (3x-4)(3x+4) \\
 = (3x)^2 - (4)^2 \\
 = \boxed{9x^2 - 16}
 \end{array}
 \quad
 \begin{array}{l}
 (a^2+7b)(a^2-7b) \\
 = (a^2)^2 - (7b)^2 \\
 = \boxed{a^4 - 49b^2}
 \end{array}
 \quad
 \begin{array}{l}
 2(6q-1)(6q+1) \\
 = 2((6q)^2 - (1)^2) \\
 = 2(36q^2 - 1) \\
 = \boxed{72q^2 - 2}
 \end{array}$$

Ex. Factor:

$$\begin{array}{l}
 x^2 - 81 \\
 \boxed{(x-9)(x+9)}
 \end{array}
 \quad
 \begin{array}{l}
 -16c^2 + 25a^2 \\
 = 25a^2 - 16c^2 \\
 = \boxed{(5a-4c)(5a+4c)}
 \end{array}
 \quad
 \begin{array}{l}
 m^2 + 16 \\
 \boxed{\text{prime}}
 \end{array}
 \quad
 \begin{array}{l}
 7g^3h^2 - 28g^5 \quad \text{GCF!} \\
 = 7g^3(h^2 - 4g^2) \\
 = \boxed{7g^3(h+2g)(h-2g)}
 \end{array}$$

Ex. Factor:

$$\begin{array}{l}
 4x^2 + 12x + 9 \\
 4x^2 + 6x + 6x + 9 \\
 \underline{4x^2 + 6x + 6x + 9} \\
 2x(2x+3) + 3(2x+3) \\
 (2x+3)(2x+3) \\
 = \boxed{(2x+3)^2}
 \end{array}
 \quad
 \begin{array}{r}
 \otimes 36 \\
 \oplus 12 \\
 \hline
 6, 6
 \end{array}
 \quad
 \begin{array}{l}
 x^2 - 8x + 16 \\
 (x-4)(x-4) \\
 = \boxed{(x-4)^2}
 \end{array}
 \quad
 \begin{array}{r}
 \otimes 16 \\
 \oplus -8 \\
 \hline
 -4, -4
 \end{array}$$

Perfect Square Trinomial: The result of a binomial square

$$(a+b)^2 = a^2 + \underline{2ab} + b^2$$

$$(a-b)^2 = a^2 - \underline{2ab} + b^2$$

Ex. Multiply:

$$(x+5)^2 = \boxed{x^2 + 10x + 25}$$

$$(3q-2)^2 = \boxed{9q^2 - 12q + 4}$$

$$\begin{aligned} 3(2a+c)^2 &= 3(4a^2 + 4ac + c^2) \\ &= \boxed{12a^2 + 12ac + 3c^2} \end{aligned}$$

How can we identify a perfect square trinomial? ✓

perfect square $x^2 + 10x + 25$

perfect square $9q^2 - 12q + 4$

Diagram for $x^2 + 10x + 25$:
 $\sqrt{x^2} = x$, $\sqrt{25} = 5$, $2(x)(5) = 10x$

Diagram for $9q^2 - 12q + 4$:
 $\sqrt{9q^2} = 3q$, $\sqrt{4} = 2$, $2(3q)(2) = 12q$

Ex. Determine the values of q that would make each trinomial a perfect square:

$$\begin{aligned} x^2 + qx + 1 \\ \sqrt{x^2} = x, \sqrt{1} = 1 \\ 2(x)(1) = 2x \\ q = 2 \\ = (x+1)^2 \end{aligned}$$

$$\begin{aligned} 9x^2 + qx + 16 \\ \sqrt{9x^2} = 3x, \sqrt{16} = 4 \\ 2(3x)(4) = 24x \\ q = 24 \\ = (3x+4)^2 \end{aligned}$$

$$\begin{aligned} 49a^2 + qab + 36b^2 \\ \sqrt{49a^2} = 7a, \sqrt{36b^2} = 6b \\ 2(7a)(6b) = 84ab \\ q = 84 \\ = (7a+6b)^2 \end{aligned}$$

Ex. Determine if each trinomial is a perfect square, if yes factor using the pattern:

$$\begin{aligned} x^2 + 6x + 9 \\ \sqrt{x^2} = x, \sqrt{9} = 3 \\ 2(x)(3) = 6x \\ = (x+3)^2 \end{aligned}$$

$$\begin{aligned} x^2 - 6x + 9 \\ = (x-3)^2 \end{aligned}$$

$$\begin{aligned} c^2 - 14c + 36 \\ \sqrt{c^2} = c, \sqrt{36} = 6 \\ 2(c)(6) = 12c \neq 14c \\ \text{not perfect square} \end{aligned}$$

prime

$$\begin{aligned} 25p^2 - 40pq + 16q^2 \\ \sqrt{25p^2} = 5p, \sqrt{16q^2} = 4q \\ 2(5p)(4q) = 40pq \\ = (5p-4q)^2 \end{aligned}$$

$$\begin{aligned} 25p^2 + 40pq + 16q^2 \\ = (5p+4q)^2 \end{aligned}$$

$$\begin{aligned} 4x^2 + 10x + 25 \\ \sqrt{4x^2} = 2x, \sqrt{25} = 5 \\ 2(2x)(5) = 20x \neq 10x \\ \text{not perfect square} \end{aligned}$$

prime

$$\begin{aligned} x^2 + 100 \\ x^2 + 10 \\ \text{not perfect square} \end{aligned}$$