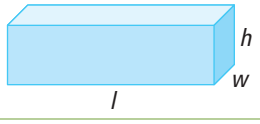
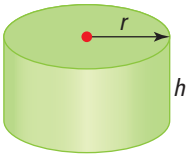
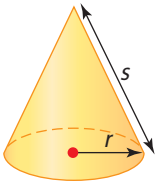
5.2 Common Factors

Focus on ...

- determining prime factors, greatest common factors, and least common multiples of whole numbers
- writing polynomials in factored form
- applying your understanding of factors and multiples to solve problems

Cubism is an early 20th-century art style. It was pioneered by artists Pablo Picasso and Georges Braque. In cubist artworks, natural forms are broken up. The pieces are reassembled into simplified 3-D shapes. The idea is to portray an object from multiple points of view at the same time. The painting shown is Picasso's *Factory, Horta de Ebbo* (1909).

When calculating the surface area of a 3-D shape, the same formula can often be used in different ways. For example, the formula for the surface area of a right prism, a right cylinder, or a right cone can be written in two forms:

Shape	Formula #1	Formula #2
Right prism 	$SA = 2lw + 2lh + 2wh$	$SA = 2(lw + lh + wh)$
Right cylinder 	$SA = 2\pi r^2 + 2\pi rh$	$SA = 2\pi r(r + h)$
Right cone 	$SA = \pi r^2 + \pi rs$	$SA = \pi r(r + s)$

Compare the two surface area formulas for each shape. What is similar about the two formulas? What is different about the two formulas? Is there one surface area formula for each shape that you prefer to use? Explain.

Investigate Common Factors

1. a) Write the number 30 as a product of prime factors. How do you know the factors are prime?

b) Can you write the number 1 as a product of prime factors? Explain why or why not.

c) Can you write the number 0 as a product of prime factors? Explain why or why not.

2. Write each of the following pairs of numbers as a product of prime factors. Identify the **greatest common factor (GCF)** of each pair.

a) 60 and 48 **b)** 25 and 40

c) 16, 24, and 36

3. Identify the **least common multiple (LCM)** of each pair of numbers.

a) 12 and 15 **b)** 20 and 25

c) 18 and 32

4. a) What is the GCF of 72 and 48?

b) Write each number as the product of two factors, where one factor is the GCF.

c) Explain how you determined the second factor.

5. a) Identify the GCF of each pair of terms.

6^2 and 6^3 8^4 and 8^7 x^5 and x^2

b) Compare the methods you used to identify the GCF of whole numbers and the GCF of variable terms. What are the similarities and differences between the methods?

6. a) Identify the GCF of x^5 and x^7.

b) Write each term as the product of two factors, where one factor is the GCF.

c) Explain how you determined the second factor.

7. a) Identify the GCF of the polynomial $12x^4 + 8x^3$.

How would writing each term as a product help?

b) Rewrite the polynomial as the sum of products. Express each term as a product of two factors, where the first factor is the GCF.

c) Explain how you determined the second factor.

8. Reflect and Respond Explain how to factor a polynomial using the GCF.

greatest common factor (GCF)

- the largest factor shared by two or more terms
- for example, the GCF of 12 and 28 is 4

least common multiple (LCM)

- the smallest multiple shared by two or more terms
- Multiples of 6 and 8 are 24, 48, 72, ... The LCM is 24.

Link the Ideas

Factor out the GCF from a polynomial by dividing each term by the GCF. Then, the polynomial can be written in a simpler form to solve more complex problems.

$15x^2 + 10x = 5x(3x + 2)$

Example 1 Determine the Greatest Common Factor

Determine the GCF of $16x^2y$ and $24x^2y^3$.

Solution

Method 1: Use Prime Factorization

List the prime factorization of the numerical coefficients.

$16 = (2)(2)(2)(2)$

$24 = (2)(2)(2)(3)$

Numerical GCF $= (2)(2)(2)$
$= 8$

List the prime factorization of the variables.

$x^2y = (x)(x)(y)$

$x^2y^3 = (x)(x)(y)(y)(y)$

Variable GCF $= (x)(x)(y)$
$= x^2y$

Therefore, the GCF of $16x^2y$ and $24x^2y^3$ is $8x^2y$.

Method 2: List the Factors

Write the factors of each term.

$16x^2y$: 1, 2, 4, 8, 16, x, x^2, y

$24x^2y^3$: 1, 2, 3, 4, 6, 8, 12, 24, x, x^2, y, y^2, y^3

The greatest common factors are 8, x^2, and y.

Therefore, the GCF of $16x^2y$ and $24x^2y^3$ is $8x^2y$.

Your Turn

Determine the GCF of each pair of terms.

a) $5m^2n$ and $15mn^2$

b) $48ab^3c$ and $36a^2b^2c^2$

Did You Know?

For *prime factorization*, write a number as the product of prime numbers. For example, the prime factorization of 18 is $2 \times 3 \times 3$.

Example 2 Write a Polynomial in Factored Form

Write $7a^2b - 28ab + 14ab^2$ in factored form.

Solution

Identify the GCF of the numerical coefficients by listing the prime factorization for each coefficient.

$7 = (7)$

$28 = (2)(2)(7)$

$14 = (2)(7)$

The GCF is 7.

Identify the GCF of the variables.

$a^2b = (a)(a)(b)$

$ab = (a)(b)$

$ab^2 = (a)(b)(b)$

The GCF is ab.

Therefore, the GCF of $7a^2b - 28ab + 14ab^2$ is $7ab$.

Divide each term by the GCF.

$7a^2b - 28ab + 14ab^2 = 7ab(a - 4 + 2b)$

Check:

Multiply.

$$\begin{aligned} 7ab(a - 4 + 2b) &= (7ab)(a) + (7ab)(-4) + (7ab)(2b) \\ &= 7a^2b - 28ab + 14ab^2 \end{aligned}$$

Multiplying is the reverse of factoring.

Your Turn

Write each polynomial in factored form.

a) $27r^2s^2 - 18r^3s^2 - 36rs^3$

b) $4np^2 + 10n^4p - 12n^3p$

Example 3 Determine Binomial Factors

Write each expression in factored form.

a) $3x(x - 4) + 5(x - 4)$

b) $y^2 + 8xy + 2y + 16x$

Solution

a) The GCF can be a binomial expression. The GCF for the terms $3x(x - 4)$ and $5(x - 4)$ is $(x - 4)$.

$3x(x - 4) + 5(x - 4) = (x - 4)(3x + 5)$

b) A GCF may be found by grouping terms.

$$\begin{aligned} y^2 + 8xy + 2y + 16x &= (y^2 + 8xy) + (2y + 16x) \\ &= y(y + 8x) + 2(y + 8x) \\ &= (y + 2)(y + 8x) \end{aligned}$$

Check:

Multiply.

$$\begin{aligned} (y + 2)(y + 8x) &= y(y + 8x) + 2(y + 8x) \\ &= y^2 + 8xy + 2y + 16x \end{aligned}$$

Your Turn

Write each expression in factored form.

a) $4(x + 5) - 3x(x + 5)$

b) $a^2 + 8ab + 2a + 16b$

Example 4 Using the Greatest Common Factor to Solve a Problem

Paula has 18 toonies, 30 loonies, and 48 quarters. She wants to group her money so that each group has the same number of each coin and there are no coins leftover.

a) What is the maximum number of groups she can make?

b) How many of each coin will be in each group?

c) How much money will each group be worth?

Solution

a) To find the maximum number of groups, identify the GCF of 18, 30, and 48.
The factors of 18 are 1, 2, 3, 6, 9, and 18.
The factors of 30 are 1, 2, 3, 5, 6, 10, 15, and 30.
The factors of 48 are 1, 2, 3, 4, 6, 8, 12, 16, 24, and 48.
The GCF is 6. Therefore, the maximum number of groups is 6.

b) Divide each number of coins by the GCF.
$\frac{18}{6} = 3$ $\frac{30}{6} = 5$ $\frac{48}{6} = 8$
There will be 3 toonies, 5 loonies, and 8 quarters in each group.

c) Multiply the number of each coin by its value.
Toonies: (3)($2) = $6
Loonies: (5)($1) = $5
Quarters: (8)($0.25) = $2
6 + 5 + 2 = 13
Each group will have a value of $13.

Your Turn

The students in Mr. Noyle's Construction class have decided they want to build dog houses for their class project. The class will be split up into groups. Each group will construct their dog house with the same type and amount of lumber. Mr. Noyle has 24 ten foot 1 by 4s, 32 eight foot 2 by 4s, and 8 sheets of plywood (4′ by 8′) available to use for this project.

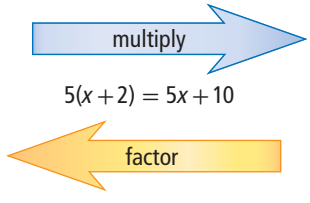
a) What is the maximum number of groups of students that can build dog houses?

b) How much of each lumber type will each group have to work with?

c) What is the total length of 2 by 4s and 1 by 4s that each group will have to work with?

Key Ideas

- Factoring is the reverse of multiplying.



- To find the GCF of a polynomial find the GCF of the coefficients and variables.
- To factor a GCF from a polynomial divide each term by the GCF.
- Polynomials can be written as a product of the GCF and the sum or difference of the remaining factors.

  $2m^3n^2 - 8m^2n + 12mn^4 = 2mn(m^2n - 4m + 6n^3)$
- A common factor can be any polynomial, such as a binomial.

  $a(x + 2) - b(x + 2)$ has a common factor of $x + 2$.

Check Your Understanding

Practise

1. Copy the table. List all of the factors of each pair of numbers. Then, identify the greatest common factor (GCF).

a) 20: 30: GCF:	**b)** 28: 40: GCF:
c) 30: 48: GCF:	**d)** 36: 27: GCF:

2. Identify the GCF of the following sets of numbers.

a) 48 and 36

b) 144 and 96

c) 81 and 54

d) 256, 216, and 78

e) 50, 100, and 625

3. Identify the least common multiple (LCM) of the following sets of numbers.

a) 12 and 16

b) 15 and 20

c) 18 and 30

d) 10, 15, and 25

e) 22, 33, and 44

4. Identify the GCF of the following sets of terms.

a) $15a^2b$ and $18ab$

b) $27m^2n^3$ and $81m^3n$

c) $8x^2y^2$ and $24x^3y^3$

d) $12a^3bc^2$, $28a^2c$, and $36a^2b^2c^2$

e) $14p^4q^5$, $-24p^5q^4$, and $7p^3q^3$

5. Factor the following polynomials.

a) $5x + 15$

b) $3y^2 - 5y$

c) $w^2x + w^2y - w^2z$

d) $6a^3b - 18ab^2$

e) $9x^3 - 12x^2 + 6x$

6. State the missing factor.

a) $6a^2bc + 9ab^2 = (\blacksquare)(2ac + 3b)$

b) $3s^2 - 15 = 3(\blacksquare)$

c) $3d^2 - 21d = 3d(\blacksquare)$

d) $16x^2 - 2x = 2x(\blacksquare)$

e) $12x^2y^2 - 16xy = (\blacksquare)(3xy - 4)$

7. Factor the following polynomials.

a) $3y(y - 2) + 4(y - 2)$

b) $5a(a - 4) - 2(a - 4)$

c) $2cx - 8x + 7c - 28$

d) $3x^2 - 9x - 8x + 24$

e) $2y^4 + y^3 - 10y - 5$

Apply

8. Mei is stacking toy blocks that are 12 cm tall next to blocks that are 18 cm tall. What is the shortest height at which the two stacks will be the same height?

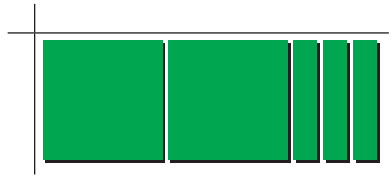
9. Explain the difference between listing the factors of a number and listing the multiples of a number.

10. The models show rectangles of algebra tiles. Answer the following questions for each rectangle.

- What expression does each model represent?
- What are the possible dimensions that could produce each rectangle?
- Write an expression for each model, using your dimensions.

a)

b)



11. **a)** Write a polynomial with two terms that have a GCF of $6x$.

b) Write a polynomial with two terms that have a GCF of $4ab$.

c) Write a polynomial with three terms that have a GCF of $2m^3n^2$.

12. Each of the following factored polynomials has an error or is not fully factored. Describe what needs to be fixed and correct each one.

a) $15x^2 - 3x = 3x(5x - 0)$

b) $5x(x - 2) - (x - 2) = (x - 2)(5x)$

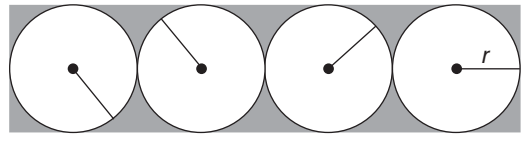
c) $9a^2b^3 - 27a^2b^2 + 81a^3b^3 = 9ab(ab^2 - 3ab + 9a^2b^2)$

d) $4fx + 16f + 2x + 8 = 2f(2x + 8) + 1(2x + 8)$

e) $2p^2 - 20p + 6p - 10 = 2p(p - 10 - 3) - 10$
$= 2p(p - 23)$

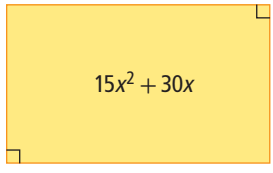
13. Nikolai has 30 pencils, 48 pens, and 36 erasers. He needs to package these items in containers for the participants of a workshop he is giving. He wants to divide them into identical containers, so that each container has the same number of each of the pencils, pens, and erasers. If he wants each container to have the greatest number of items possible, how many plastic containers does he need?

14. Some natural gas meters have four dials to show the gas use. Write a factored expression to represent the area of the metal plate around the dials, shaded in grey.



15. Mario wants to cut two pieces of material into equal-size squares with no material wasted. One piece measures 12 in. by 36 in. The other measures 6 in. by 42 in. What is the largest size square that he can cut?

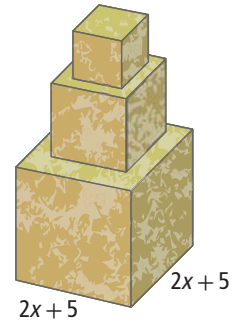
16. A rectangle has an area that can be represented by the expression $15x^2 + 30x$. The length and width can be found by factoring the expression. Write possible expressions for the length and the width.



Extend

17. The greatest common factor of two numbers is 871. Both numbers are even. Neither is divisible by the other. What are the smallest two numbers they could be?

18. The pedestal design is made up of square-based prisms. The base length of each prism is 3 cm less than that of the layer immediately below.



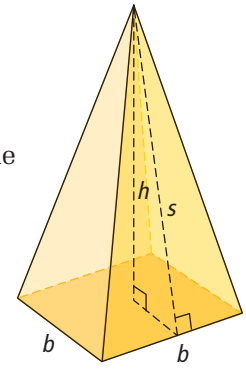
a) Write an algebraic expression for the total top surface area of the three prisms used to make the pedestal.

b) Multiply and then simplify.

c) Factor the expressions from part b).

Create Connections

19. The surface area of a square-based pyramid is $SA = b^2 + 2bs$.



a) Write the formula in factored form.

b) Use both forms of the formula to calculate the surface area when $b = 5$ cm and $s = 4$ cm.

c) What is the same about each surface area you calculated? What is different about each surface area you calculated?

d) Is there one surface area formula you prefer to use? Explain.