5.1 Multiplying Polynomials

Focus on ...

- multiplying polynomials
- explaining how multiplication of binomials is related to area and to the multiplication of two-digit numbers

Geometric abstraction is a form of abstract art based on the use of geometric shapes. Piet Mondrian is one of many artists who used this style of painting. Mondrian's art has influenced designers of everything from cups to buildings.

In what ways can you relate **polynomial** multiplication to Mondrian's painting?

polynomial

- a sum of monomials
- for example, $x + 5$, $2a^2 - 6ab + 18b^2$

Investigate Multiplying Polynomials

Materials

- algebra tiles

You can use algebra tiles to model algebraic expressions.

positive x-tile

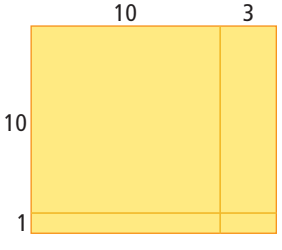
positive x^2-tile

positive 1-tile

The same tiles in white represent negative quantities.

1. Complete the multiplication (13)(11).

2. a) You can express 13 as the sum $10 + 3$, and 11 as the sum $10 + 1$. Complete an area model with the dimensions $10 + 3$ and $10 + 1$.

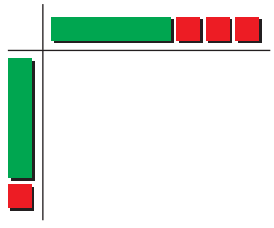


b) How does your model show the factors $10 + 3$ and $10 + 1$?

c) What is the product?

3. Compare the methods in step 1 and step 2. How are they similar? How are they different?

4. a) Use algebra tiles to determine the product $(x + 3)(x + 1)$. Sketch your model.



b) How does your model show the factors $x + 3$ and $x + 1$?

c) What is the product?

d) How is your model similar to and different from the model you used in step 2?

5. Use algebra tiles to determine each product.

a) $(x + 5)(x - 2)$

b) $(p + 4)(p + 4)$

c) $(2x + 4)(3x - 1)$

6. Reflect and Respond Discuss the following questions with a partner.

a) Use your answers from step 5 to look for patterns that relate the factors to the products.

b) Use the patterns you found to explain how you could multiply **binomials** without using algebra tiles.

binomial

- a polynomial with two terms
- for example, $x + 3$, $2x - 5y$

Link the Ideas

You can make connections between multiplying whole numbers and multiplying polynomials. Multiply 42 by 26 using the **distributive property**.

distributive property

- the rule that states $a(b + c) = ab + ac$
- for example, $40(20 + 6) = (40)(20) + (40)(6)$

$$\begin{aligned}(42)(26) &= (40 + 2)(20 + 6)\\ &= 40(20 + 6) + 2(20 + 6)\\ &= (40)(20) + (40)(6) + (2)(20) + (2)(6)\\ &= 800 + 240 + 40 + 12\\ &= 1092\end{aligned}$$

Example 1 Multiply Binomials

Multiply.

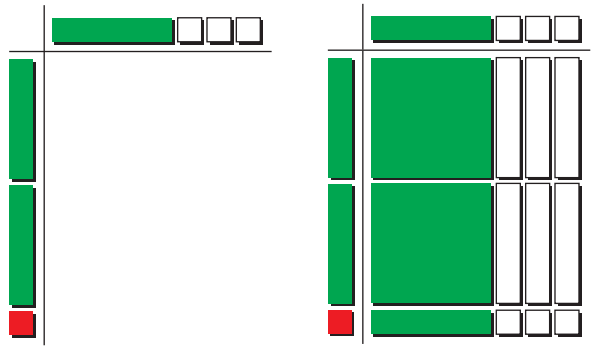
a) $(x - 3)(2x + 1)$ **b)** $(x - 2y)(x - 4y)$

Solution

a) Method 1: Use Algebra Tiles

Use algebra tiles to show the dimensions $x - 3$ and $2x + 1$. Then, complete a rectangle that has these dimensions.

To create an area model, you place the negative tiles on top of the positive tiles. Explain why.



There are two x^2-tiles, six negative x-tiles, one positive x-tile, and three negative 1-tiles in the rectangle.

Therefore, $(x - 3)(2x + 1) = 2x^2 - 5x - 3$.

Method 2: Use the Distributive Property

Multiply each term in the first binomial by each term in the second binomial. Then, combine like terms.

$$\begin{aligned}(x - 3)(2x + 1) &= x(2x + 1) - 3(2x + 1)\\ &= (x)(2x) + (x)(1) + (-3)(2x) + (-3)(1)\\ &= 2x^2 + 1x - 6x - 3\\ &= 2x^2 - 5x - 3\end{aligned}$$

b) $$\begin{aligned}(x - 2y)(x - 4y) &= x(x - 4y) - 2y(x - 4y)\\ &= x^2 - 4xy - 2xy + 8y^2\\ &= x^2 - 6xy + 8y^2\end{aligned}$$

Check:

You can verify your work by substituting values for the variables x and y. For example, substitute $x = 5$ and $y = 1$.

Left Side	Right Side
$(x - 2y)(x - 4y)$	$x^2 - 6xy + 8y^2$
$= [5 - 2(1)][5 - 4(1)]$	$= (5)^2 - 6(5)(1) + 8(1)^2$
$= (5 - 2)(5 - 4)$	$= 25 - 30 + 8$
$= (3)(1)$	$= 25 - 22$
$= 3$	$= 3$

Left Side = Right Side

Your Turn

Determine each product.

a) $(x - 3)(x - 5)$ **b)** $(5m - 1)(2m - 6)$

Example 2 Multiply a Binomial and a Trinomial

Multiply the following binomial and **trinomial**.
$(x + 2)(2x^2 - 5x + 1)$

trinomial
- a polynomial with three terms
- for example, $x^2 + 3x - 1$, $2x^2 - 5xy + 10y^2$

Solution

$$\begin{aligned}
&(x + 2)(2x^2 - 5x + 1) \\
&= x(2x^2 - 5x + 1) + 2(2x^2 - 5x + 1) \\
&= 2x^3 - 5x^2 + x + 4x^2 - 10x + 2 \\
&= 2x^3 - x^2 - 9x + 2
\end{aligned}$$

Multiply each term in the binomial by each term in the trinomial. Then, combine like terms.

Your Turn

Determine each product.

a) $(r - 4)(3r^2 + 8r - 6)$

b) $(5x - 3)(2x^2 - 6x + 12)$

Example 3 Perform Operations on Products of Polynomials

Simplify.

a) $(x + 1)(5x + 3) + 3(2x + 4)(6x - 2)$

b) $(3w - 2)(4w + 5) - (w - 7)(2w + 3)$

Solution

a)
$$\begin{aligned}
&(x + 1)(5x + 3) + 3(2x + 4)(6x - 2) \\
&= x(5x + 3) + 1(5x + 3) + 3[2x(6x - 2) + 4(6x - 2)] \\
&= 5x^2 + 3x + 5x + 3 + 3(12x^2 - 4x + 24x - 8) \\
&= 5x^2 + 3x + 5x + 3 + 36x^2 - 12x + 72x - 24 \\
&= 41x^2 + 68x - 21
\end{aligned}$$

When you have three factors, you can multiply in any order. What are some other ways you could multiply $3(2x + 4)(6x - 2)$?

b)
$$\begin{aligned}
&(3w - 2)(4w + 5) - (w - 7)(2w + 3) \\
&= (3w)(4w + 5) + (-2)(4w + 5) - [w(2w + 3) - 7(2w + 3)] \\
&= 12w^2 + 15w - 8w - 10 - (2w^2 + 3w - 14w - 21) \\
&= 12w^2 + 15w - 8w - 10 - 2w^2 - 3w + 14w + 21 \\
&= 10w^2 + 18w + 11
\end{aligned}$$

Your Turn

Multiply and then combine like terms.

a) $(x + 3)(5x - 2) + 4(x - 1)(2x + 5)$

b) $2(3x - 2) - (4x + 7)(2x - 5)$

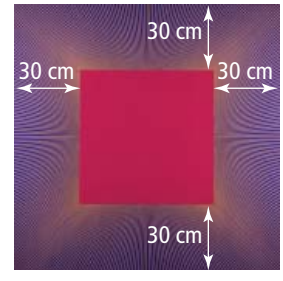
Example 4 Apply Binomial Multiplication

The painting shown is *Deep Magenta Square* by Richard Anuszkiewicz. It can be used to represent binomial multiplication. The length of the red square in the painting is unknown. The width of the border around the square is 30 cm.

a) What polynomial expression represents the total area of the painting?

b) What is the total area of the painting if the red square has an area of 3600 cm^2?

Solution



a) Let x represent the length of the red square.
The length of the painting can be represented by $x + 30 + 30 = x + 60$.
The area of the painting can be represented by the polynomial expression $(x + 60)(x + 60) = x^2 + 120x + 3600$.

b) If the red square has an area of 3600 cm^2, the side length of the red square is $\sqrt{3600} = 60$. Substitute this value into the expression $(x + 60)(x + 60)$ or the expression $x^2 + 120x + 3600$.

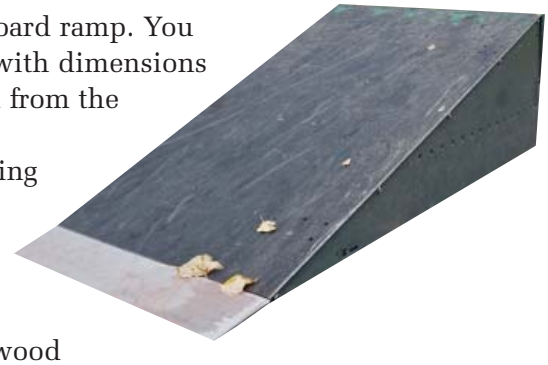
$$(60 + 60)(60 + 60) = (120)(120)$$
$$= 14\,400$$

or

$$(60)^2 + 120(60) + 3600 = 3600 + 7200 + 3600$$
$$= 14\,400$$

The area of the painting is 14 400 cm^2.

Your Turn



You are building a skateboard ramp. You have a piece of plywood with dimensions of 4 ft by 8 ft. You cut x ft from the length and width.

a) Sketch a diagram showing the cuts made to the piece of plywood. Label the dimensions.

b) What is the area of the remaining piece of plywood that will be used for the ramp?

Key Ideas

- You can use the distributive property to multiply polynomials. Multiply each term in the first polynomial by each term in the second polynomial.

$$\begin{aligned}(3x - 2)(4x + 5) &= (3x)(4x + 5) - 2(4x + 5)\\ &= 12x^2 + 15x - 8x - 10\\ &= 12x^2 + 7x - 10\end{aligned}$$

$$\begin{aligned}(c - 3)(4c^2 - c + 6) &= c(4c^2 - c + 6) - 3(4c^2 - c + 6)\\ &= 4c^3 - c^2 + 6c - 12c^2 + 3c - 18\\ &= 4c^3 - 13c^2 + 9c - 18\end{aligned}$$

Check Your Understanding

Practise

1. Multiply using algebra tiles.

a) $(x - 2)(x + 3)$

b) $(3x - 4)(2x - 1)$

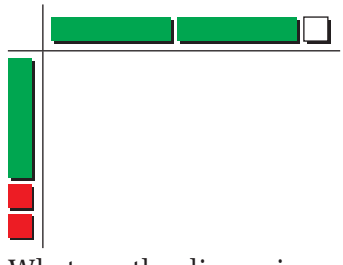
c) $(x - 5)(x - 2)$

d) $(x + 3)^2$ $(x + 3)^2$ means $(x + 3)(x + 3)$.

e) $(x + 4)(x + 7)$

f) $(2x - 5)(x - 3)$

2. a) What product does the algebra tile model show?



b) What are the dimensions of the model?

3. Multiply using the distributive property.

a) $(x + 5)(x - 2)$

b) $(x - 3)^2$

c) $(c - d)(c + d)$

d) $(4x + y)(x + y)$

e) $(y + 3)^2$

f) $(4j + 2k)(6j - 3k)$

4. Use the distributive property to determine each product.

a) $x(3x^2 - 5x + 8)$

b) $a(7b^2 + b - 1)$

c) $(x - 3)(6x^2 - 4x - 12)$

d) $(2x - 1)(5x^2 + 4x - 5)$

e) $(4s^2 + s)(3s^2 - 2s + 6)$

f) $(2y^2 + 3y - 1)(y^2 + 4y + 5)$

5. Match each binomial multiplication on the left with a trinomial on the right.

a) $(x + 1)(x - 2)$

b) $(x - 3)(x - 4)$

c) $(x - 1)^2$

d) $(x + 4)(x - 3)$

e) $(x - 3)(x - 5)$

f) $(x + 3)^2$

g) $(x + 9)(x + 4)$

h) $(x - 6)(x - 3)$

A $x^2 + 13x + 36$

B $x^2 - x - 2$

C $x^2 - 2x - 1$

D $x^2 + x - 12$

E $x^2 + 6x + 9$

F $x^2 - 2x + 1$

G $x^2 - 9x + 18$

H $x^2 - 7x + 12$

I $x^2 - 7x - 12$

J $x^2 - 8x + 15$

6. Multiply. Then, combine like terms.

a) $(4n + 2) + (2n - 3)(3n - 2)$

b) $(f + 7)(2f - 4) - (3f + 1)^2$

c) $(b - 2d)(5b - 3d) + (b + d)(4b + d)$

d) $(4x - 2)(3x - 5) + 2(7x + 5)(2x - 6)$

e) $3(5a + 3c)(2a - 3c) - (4a + c)^2$

f) $(y^2 - 5y - 6)(4y^2 + 6y + 1)$

Apply

7. The painting shown is by Métis artist Leah Marie Dorion from Prince Albert, Saskatchewan. It is called *Hawk Woman* (2006). The frame is 2 in. wide on each side of the square painting. Write an expression to represent the dimensions and area of the painting. Multiply, and then combine like terms.

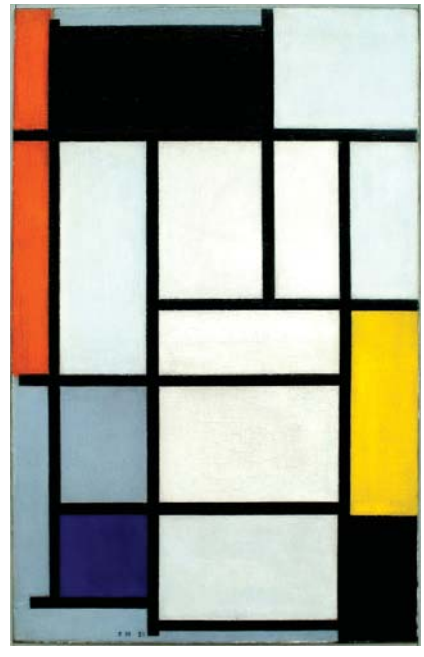
8. **Unit Project** Sketch an area model or an algebraic model to represent each multiplication. Use specific polynomials for each multiplication. Label your diagrams. Then, write the result of each multiplication as an equation.

a) (monomial)(binomial)

b) (binomial)(binomial)

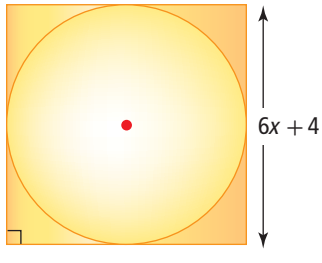
c) (binomial)(trinomial)

9. **Unit Project** Use an arrangement of algebra tiles to show combining like terms of polynomials. Arrange them artistically. Use the style of Piet Mondrian's paintings, shown here and on page 204. Write the corresponding algebraic equation that summarizes your result.



10. Darien trimmed a square photo to fit into a rectangular frame. He cut 7 cm from one side and 4 cm from the other. Let x represent the side length of the original square photo. Write an expression for the area of the trimmed photo. Multiply, and then combine like terms.

11. A circle is inset into a square with a side length of $6x + 4$, as shown. Write an expression to represent the area of the circle. Multiply, and then combine like terms.



12. Bryan was asked to multiply two binomials. He completed the following work.

$(p + 3)(p + 7) = p^2 + 7p + 3p + 21$ Step 1
$= p^2 + 10p + 21$ Step 2
$= 11p^2 + 21$ Step 3

a) Is Bryan's work correct? If not, which step is incorrect?

b) Choose any number for p. Determine whether the following equation is true.
$(2p - 3)(p + 4) = 2p^2 - 5p - 12$

13. The Li family has a house with a length of 13 m and a width of 9 m. Due to lot restrictions, they can make an addition of only y metres to the width and x metres to the length.

a) Sketch a diagram of the area of the house. Label the dimensions.

b) Write an expression for the area of the house, including the addition.

c) Calculate the area if $x = 1$ m and $y = 2$ m.

Did You Know?

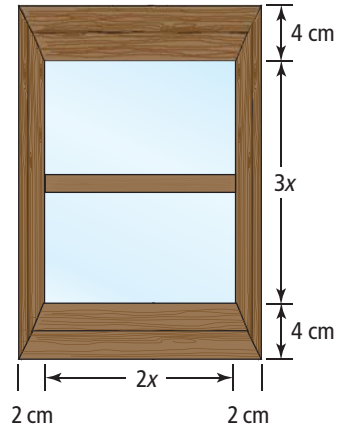
Authentic oriental rugs are hand woven and knotted. They are produced primarily in Asia, the Middle East, and India. The rugs are made for decorative, practical, and sometimes spiritual purposes.

14. a) Susan is making a rectangular area rug with a similar design to the square rug she made earlier. What are the dimensions if the new rug is 2 ft longer and 1 ft narrower than the square rug?

b) Write an expression for the area of the new rug.

c) If the square rug is 3 ft by 3 ft, which rug has the greater area? Show your work.

15. Vera is installing a kitchen window that has a height-to-width ratio of 3 : 2. The window frame adds 4 cm to the width and 8 cm to the height.



a) Write a polynomial expression that represents the total area of the window, including the frame. Multiply and combine like terms.

b) Calculate the area when $x = 12$ cm.

16. André multiplied the expression $(2x - 4)(3x + 5)$. When he checked his answer, he discovered an error.

$$(2x - 4)(3x + 5) = 2x(3x + 5) - 4(3x + 5)$$
$$= 6x^2 + 10x - 12x + 20$$
$$= 6x^2 - 2x + 20$$

Check:
Let $x = 4$.

Left Side
$(2x - 4)(3x + 5)$
$= [2(4) - 4][3(4) + 5]$
$= (8 - 4)(12 + 5)$
$= (4)(17)$
$= 68$

Right Side
$6x^2 - 2x + 20$
$= 6(4)^2 - 2(4) + 20$
$= 96 - 8 + 20$
$= 108$

a) Explain how André knew that he had made an error.

b) Explain the error and how to correct it.

Extend

17. The average number of burgers, b, sold at The Burger Barn daily can be represented by $b = 550 - 100p$, where p is the price of a burger, in dollars.

a) How does the average number of burgers sold change as the price of a burger increases?

b) Solve the equation for p.

c) The revenue from burger sales can be represented by $R = np$, where R is the total revenue, in dollars, and n is the number of burgers sold. Substitute your expression for p from part b). Then, multiply to get an expression for the daily burger revenue.

Create Connections

18. a) Choose four consecutive whole numbers. Multiply the first and last numbers. Multiply the second and third numbers. Repeat this multiplication with several different groups of four consecutive whole numbers. What pattern to you notice?

b) Let n represent your first number. What algebraic expressions represent your second, third, and fourth numbers?

c) Use algebraic multiplication to show that your pattern in part a) is always true.

19. The product of 45 and 34 can be thought of as $(40 + 5)(30 + 4)$. You can represent $40 + 5$ as $4t + 5$, where t represents 10.

a) What expression could represent $30 + 4$?

b) Use binomial multiplication of the algebraic expression. Substitute to find the product of 45 and 34.