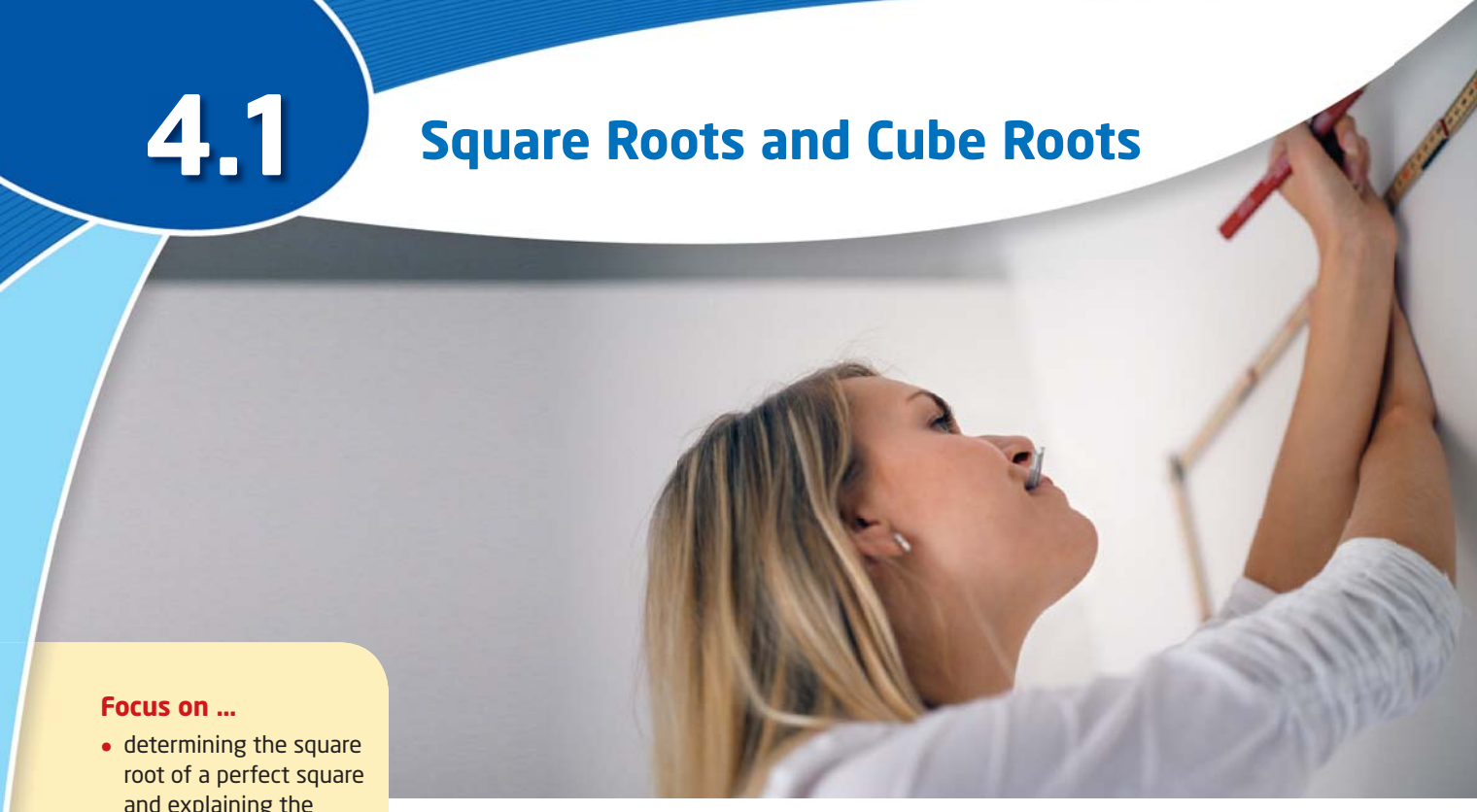
4.1 Square Roots and Cube Roots



Focus on ...

- determining the square root of a perfect square and explaining the process
- determining the cube root of a perfect cube and explaining the process
- solving problems involving square roots or cube roots

Workers apply what they know about surface area and volume when working with square shapes and cubes.

A house painter must calculate the surface area of the walls of a house when preparing a cost estimate. If you know the area of a square wall, how could you calculate the side lengths?

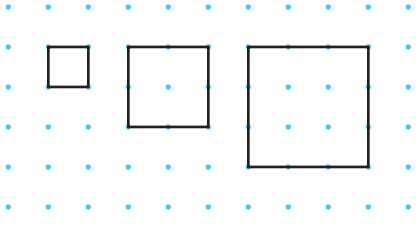
A designer must calculate the size of the case required to enclose a speaker for a sound system. If you know the volume of a cube-shaped box, how could you calculate the edge lengths?

Materials

- square dot paper
- isometric dot paper

Investigate Square Roots and Cube Roots

1. a) Determine the area of each square shown. Record the information in a table.

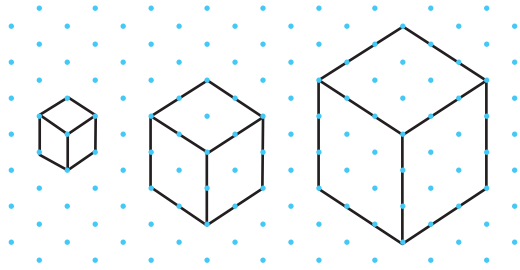


Side Length	Area in Exponential Form	Area

b) Extend the pattern for squares with dimensions of 4, 5, and 6 units.

c) What is the relationship between the side length of a square and the area of the square?

2. a) Determine the volume of each cube shown. Record the information in a table.



Edge Length	Volume in Exponential Form	Volume

b) Extend the pattern for cubes with dimensions of 4, 5, and 6 units.

c) What is the relationship between the edge length of a cube and the volume of the cube?

3. Reflect and Respond

Discuss with a partner.

a) What strategy could you use to find the side length of a square if you were given the area?

b) What strategy could you use to find the edge length of a cube if you were given the volume?

c) Explain, using a diagram, how you could predict
- the side length of a square with an area of 64 square units
- the edge length of a cube with a volume of 343 cubic units

Link the Ideas

Perfect squares and **square roots** are related to each other. The number 25 is a perfect square. It is formed by multiplying two factors of 5 together.

$(5)(5)$ or $5^2 = 25$ The symbol for square root is $\sqrt{\ }$.

The square root of 25 is 5, or $\sqrt{25} = \sqrt{(5)(5)}$
$= \sqrt{5^2}$
$= 5$

perfect square
- a number that can be expressed as the product of two equal factors
- for example, $16 = (4)(4)$ or 4^2

square root
- one of two equal factors of a number
- for example, $\sqrt{49} = \sqrt{(7)(7)}$
$= 7$

perfect cube

- a number that is the product of three equal factors
- for example, $64 = (4)(4)(4)$ or 4^3

cube root

- one of three equal factors of a number
- for example, $\sqrt[3]{512} = \sqrt[3]{(8)(8)(8)}$
  $= 8$

Perfect cubes and **cube roots** are related to each other. The number 27 is a perfect cube. It is formed by multiplying three factors of 3 together.

$(3)(3)(3)$ or $3^3 = 27$ The symbol for cube root is $\sqrt[3]{\ }$.

The cube root of 27 is 3, or $\sqrt[3]{27} = \sqrt[3]{(3)(3)(3)}$
$= \sqrt[3]{3^3}$
$= 3$

Some numbers are both perfect squares and perfect cubes.

$64 = (8)(8)$ and $64 = (4)(4)(4)$
$= 8^2$ $= 4^3$

Therefore, 64 is a perfect square and a perfect cube.

Example 1 Identify Perfect Squares and Perfect Cubes

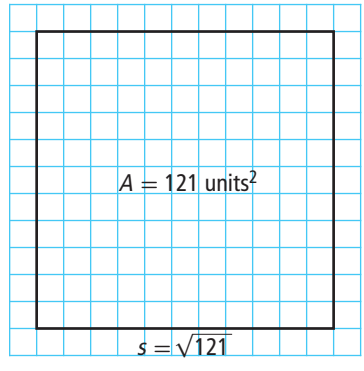
State whether each of the following numbers is a perfect square, a perfect cube, both, or neither.

a) 121 **b)** 729 **c)** 356

Solution

a) To decide whether 121 is a perfect square you might use a diagram.

$10^2 = 100$ Too low
$12^2 = 144$ Too high
$11^2 = 121$ Correct!



A square with side lengths of 11 units has an area of 121 units^2.

$(11)(11) = 121$.

Therefore, 121 is a perfect square.

To decide whether 121 is a perfect cube, you could use guess and check.

No whole number cubed results in a product of 121.

$4^3 = 64$ Too low
$5^3 = 125$ Too high

Therefore, 121 is not a perfect cube.

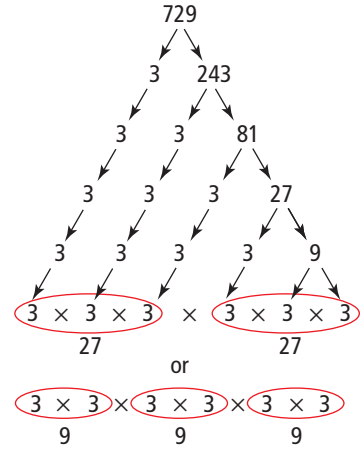
WWW Web Link

To learn more about perfect squares and square roots, go to www.mhrmath10.ca and follow the links.

To learn more about perfect cubes and cube roots, go to www.mhrmath10.ca and follow the links.

b) For 729, you might use **prime factorization**. Prime factorization involves writing a number as the product of its prime factors. A factor tree helps organize the prime factors.

Record the prime factorization for 729. Then, identify the factors that can be squared or cubed to form the product 729.



These two groups indicate the square root of 729.

These three groups indicate the cube root of 729.

You can write 729 as the product of $(27)(27) = 27^2$. Therefore, 729 is a perfect square.

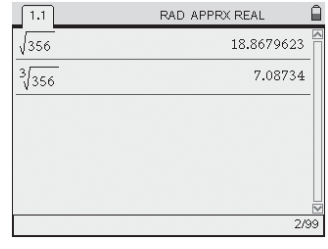
You can write 729 as the product of $(9)(9)(9) = 9^3$. Therefore, 729 is a perfect cube.

c) For 356, you might use a calculator.

C 356 $\sqrt{x}$ 18.867962

C 356 2nd $\sqrt[x]{y}$ 3 = 7.08734

Since the square root is not a whole number, 356 is not a perfect square.
Since the cube root is not an integer, 356 is not a perfect cube.
The number 356 is neither a perfect square nor a perfect cube.



Key sequences vary among calculators. Check the key sequence for determining square roots and cube roots of numbers on your calculator. Record the correct sequence for your calculator.

prime factorization

- the process of writing a number written as a product of its prime factors.
- the prime factorization of 24 is $2 \times 2 \times 2 \times 3$.

WWW Web Link

To learn more about prime factorization and to use a prime factorization tool, go to www.mhrmath10.ca and follow the links.

Did You Know?

Between 1850 and 1750 B.C.E., the Babylonians were applying the Pythagorean relationship. They recorded tables of square roots and cube roots on clay tablets. This was long before Pythagoras was born.

Your Turn

State whether each number is a perfect square, a perfect cube, both, or neither. Use a variety of methods.

a) 125 **b)** 196 **c)** 4096

Did You Know?

Canada is the largest producer of uranium in the world. It provides about one third of the world's supply. Uranium is mined mainly in Northern Ontario and Saskatchewan. The mines in Saskatchewan provide the highest grade uranium.

Example 2 Solve Problems Involving Square Roots and Cube Roots

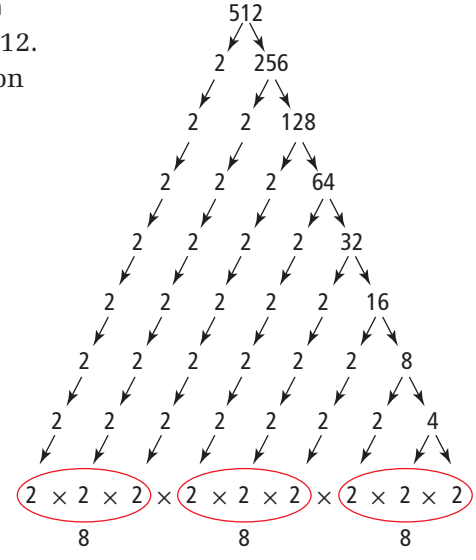
The uranium that Saskatchewan produces in a year has a volume of about 512 m^3. If this volume were made into a single cube, what would be the dimensions of the cube?

Solution

The volume of a cube of length x is given by $V = x^3$. Determine the dimensions of the cube, x, by calculating the cube root of the volume, or $x = \sqrt[3]{V}$.

Method 1: Use Prime Factorization

Determine the cube root of 512. Record the prime factorization for 512. Then, identify the factors that can be cubed to form 512.



Since there are three equal groups, you know that 512 is a perfect cube.

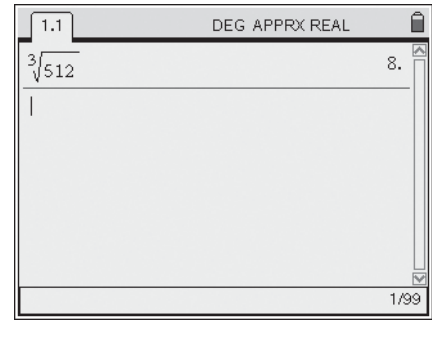
How do you know that 512 is not a perfect square?

The cube root of 512 is 8.
The cube would be 8 m in length, height, and width.

Method 2: Use a Calculator

C 512 2nd $\sqrt[x]{y}$ 3 = 8.

The cube would be 8 m in length, height, and width.



Your Turn

a) A floor mat for gymnastics is a square with an area of 196 m^2. What is its side length?

b) The volume of a cubic box is 27 000 $in.^3$ Use two methods to determine its dimensions.

Key Ideas

- A perfect square is the product of two equal factors. One of these factors is called the square root.

  36 is a perfect square: $\sqrt{36} = 6$ because $6^2 = 36$

- A perfect cube is the product of three equal factors,
  One of these factors is called the cube root.

  -125 is a perfect cube: $\sqrt[3]{-125} = -5$ because $(-5)^3 = -125$

- Numbers can be both perfect squares and perfect cubes.

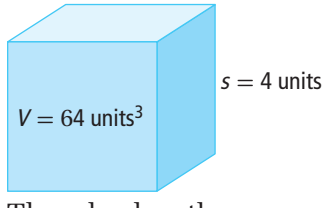
  15 625 is a perfect square: $125^2 = 15\,625$

  15 625 is a perfect cube: $25^3 = 15\,625$

- You can use diagrams or manipulatives, factor trees, or a calculator to solve problems involving square roots and cube roots.

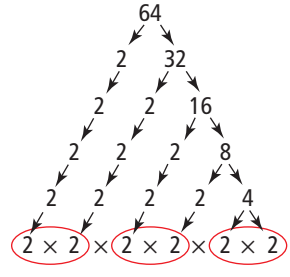
  Determine the cube root of 64.

  - Use a diagram.



The edge lengths represent the cube root:
(4)(4)(4) = 64.

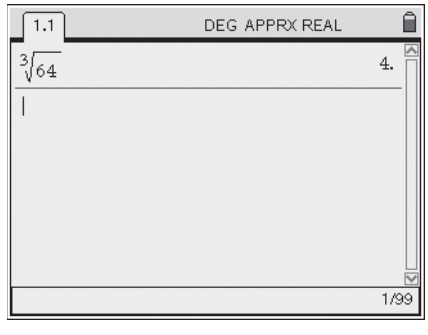
  - Use prime factorization.



There are three equal groups of 4. Therefore, the cube root of 64 is 4.

  - Use a calculator.

C 64 2nd $\sqrt[x]{y}$ 3 = 4.



Check Your Understanding

Practise

1. What is the value of each expression? Express your answers as integers or fractions.

a) 7^2 **b)** -50^2 **c)** $(-3)^2$

d) $\frac{4^2}{5}$ **e)** $\frac{3}{2^2}$ **f)** $\left(\frac{3}{4}\right)^2$

2. Evaluate. Give your answers as integers or fractions.

a) 2^3 **b)** -4^3 **c)** $(-5)^3$

d) $\frac{2^3}{4}$ **e)** $\frac{3}{6^3}$ **f)** $\left(\frac{2}{3}\right)^3$

3. What is the value of each expression?

a) $\sqrt{49}$ **b)** $\sqrt{169}$ **c)** $\sqrt{(25)(4)}$

d) $\frac{16}{\sqrt{64}}$ **e)** $\frac{\sqrt{36}}{3}$ **f)** $\sqrt{9x^2}$

4. Evaluate.

a) $\sqrt[3]{1}$ **b)** $\sqrt[3]{(8)(27)}$ **c)** $\sqrt[3]{8000}$

d) $\frac{\sqrt[3]{64}}{2}$ **e)** $\sqrt[3]{\frac{27}{125}}$ **f)** $\sqrt[3]{64a^3}$

5. Identify each number as a perfect square, a perfect cube, or both. Support your answers using a diagram or a factor tree.

a) 1 **b)** 1000 **c)** 81

d) 169 **e)** 216 **f)** 1024

6. State whether each of the following numbers is a perfect square, a perfect cube, both, or neither.

a) 144 **b)** 2197 **c)** 16

d) 225 **e)** 15 625 **f)** 117 649

7. Evaluate using prime factorization. Explain the process.

a) $\sqrt{100}$ **b)** $\sqrt[3]{8}$ **c)** $\sqrt{81}$

d) $\sqrt[3]{27}$ **e)** $\sqrt{144}$ **f)** $\sqrt{576}$

8. Calculate.

a) $\sqrt{196}$ **b)** $\sqrt[3]{4096}$ **c)** $\sqrt[3]{9261}$

d) $\sqrt[3]{3375}$ **e)** $\sqrt{961}$ **f)** $\sqrt[3]{4913}$

9. Connor needs to replace the edging on a square rug. If the rug has an area of 25 m^2, what length of edging does he need?

10. Serena collected all the garbage she created in one year. The volume of the cube it formed was 343 ft^3. What was the edge length of the cube?

Apply

11. A square wrestling mat has an area of 1444 ft^2.

a) Before calculating the side length of the mat, estimate two whole numbers between which the answer falls. Which number do you think the answer is closer to?

b) Calculate the side length.

c) How does your estimate compare to the calculated answer?

12. Star quilts are squares with a minimum area of 1 m^2 and a maximum area of 9 m^2. What are the possible whole number dimensions of such a quilt?

Did You Know?

The star quilt is a pattern used by many cultures including the Lakota, Dakota, other Sioux nations, and Europeans. It was inspired from the design for buffalo robes. When buffalo were no longer available, the star quilt replaced the buffalo robe in Aboriginal traditions.

13. **Unit Project** The mural shown below was originally created to celebrate Alberta's Centennial in 2005. It was installed at the Centre d'arts visuels de l'Alberta in Edmonton, AB. The mural symbolizes the unity of the francophone communities throughout Alberta. Your art class decides to create a mural mosaic. Your mosaic will highlight the regions of the province or territory where you live.

a) The class mosaic will be composed of 15-cm by 15-cm squares. How many squares will be needed to create a mural that covers an area of 2.7 m^2?

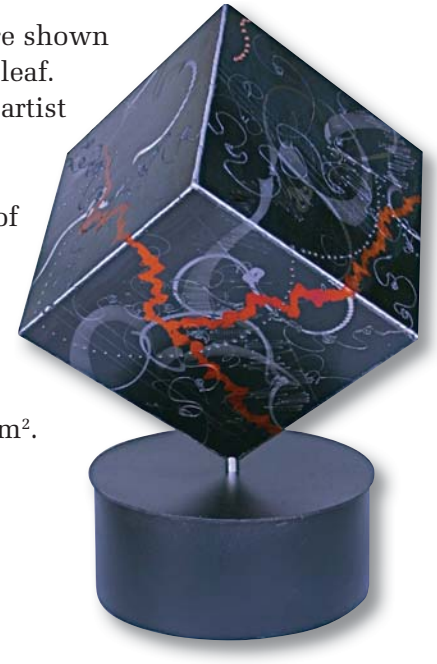
b) Design a mural to show a geometric representation of square roots.

c) How is the mural a geometric representation of square roots?

Les régions se racontent (The regions tell their story)

14. A recycling depot compresses cardboard into cubic bales. If each bale has a volume of 46 656 in.^{3}, what are its edge lengths?

15. **Unit Project** The cubic sculpture shown here is made of steel with copper leaf. It was created by Tony Bloom, an artist from Canmore, AB.



a) If it has a volume of 4913 in.^{3}, what is the length of one edge of the cube?

b) Explain how the sculpture is a geometric representation of a cube root.

16. The surface area of a die is 600 mm^2. What is the volume of the die?

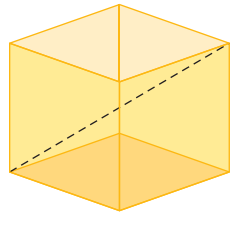
Extend

17. Meteorologists use the formula $D^3 = 684t^2$ to describe violent storms, such as tornadoes and hurricanes. D is the diameter of the storm, in kilometres, and t is the number of hours it will last.

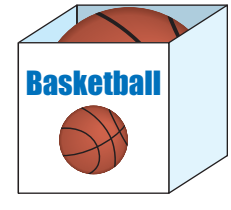
a) If a storm lasts for 4 h, what is its diameter?

b) If the diameter of a hurricane is 30 km, how long will it last?

18. A cube has a volume of 3375 cm^3. What is the diagonal distance through the cube from one corner to the opposite corner?



19. A manufacturer is designing an open, cube-shaped box to hold a basketball. The basketball has a volume of 2304π cm^3.

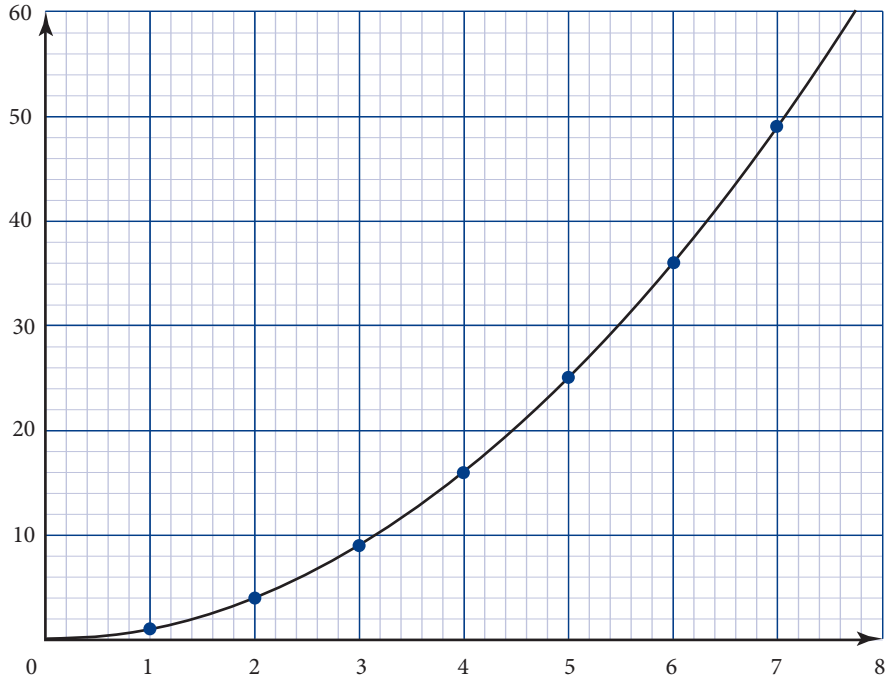


a) How much cardboard is needed to create the smallest box possible using the least amount of material? Do not include seam overlap in your calculations.

b) What is the volume of the box? What are its dimensions?

Create Connections

20. The following graph can be used to determine squares and square roots.



a) Use the graph to complete the following table of values.

Number	0	1	2	3			6	7	
Number Squared	0	1			16	25			64

b) Based on the table, how would you label the axes on the graph?

c) What does each small unit represent on the horizontal axis? vertical axis?

d) Explain how you could use the graph to find the value for 5^2.

e) How could you use the graph to evaluate $\sqrt{49}$?

f) Show how you could use the graph to determine the approximate value for $\sqrt{18}$. Multiply your answer by itself. How close is your product to 18?

g) What is an approximation for $(6.2)^2$?

21. a) Make an arithmetic question, involving a square root, that has a value of $\frac{2}{3}$.

b) Make an arithmetic question, involving a cube root, that has a value of $\frac{2}{3}$.