AP CALCULUS
VOLUME OF SOLIDS OF REVOLUTION - Extra Practice

Set up, and solve with calc., an integral for the volume of the solid obtained by rotating the region bounded by the given curves about the given line.

1. $y = x^2, x = 1, y = 0$; about the x-axis
2. $x + y = 1, x = 0, y = 0$; about the x-axis
3. $y = \sqrt{x-1}, x = 2, x = 5, y = 0$; about the x-axis
4. $y = x^2, y^2 = x$; about the x-axis
5. $y = x^2 + 1, y = 3 - x^2$; about the x-axis
6. $y^2 = x, x = 2y$; about the y-axis
7. $y = x^4, y = 1$; about $y = 2$
8. $y = x^2, y = 4, x = 0$; about:
   a) the y-axis
   b) the x-axis
   c) $y = 4$
   d) $x = 2$
   e) $x = 4$
   f) $y = 6$
   g) $x = -1$
   h) $y = -2$

Answers

1) $\pi \int_0^1 x^4 dx = \pi/5 \text{ un}^3$

2) $\pi \int_0^1 (-x+1)^2 dx = \pi/3 \text{ un}^3$

3) $\pi \int_2^5 (x-1) dx = \frac{15\pi}{2} \text{ un}^3$

4) $\pi \int_0^1 (x - x^4) dx = \frac{3\pi}{10} \text{ un}^3$

5) $\pi \int_{-1}^{1} [(3-x^2)^2 - (x^2+1)^2] dx$

OR

$2\pi \int_0^1 [(3-x^2)^2 - (x^2+1)^2] dx$

$= \frac{32\pi}{3} \text{ un}^3$

6) $\pi \int_0^2 (4y^2 - y^4) dy = \frac{64\pi}{15} \text{ un}^3$

7) $2\pi \int_0^1 \left[\left(2 - x^4\right)^2 - 1\right] dx = \frac{208\pi}{45} \text{ un}^3$

8) a. $\pi \int_0^4 y\,dy = 8\pi \text{ un}^3$

b. $\pi \int_0^2 (16 - x^4) dx = \frac{128\pi}{5} \text{ un}^3$

c. $\pi \int_0^2 (4 - x^2)^2 dx = \frac{256\pi}{15} \text{ un}^3$

d. $\pi \int_0^4 [4 - (2 - \sqrt{y})^2] dy = 41.9 \text{ un}^3$

e. $\pi \int_0^4 [16 - (4 - \sqrt{y})^2] dy = 108.9 \text{ un}^3$

f. $\pi \int_0^2 [(6 - x^2)^2 - 4] dx = \frac{192\pi}{5} \text{ un}^3$

g. $\pi \int_0^4 [(1 + \sqrt{y})^2 - 1] dy = 58.6 \text{ un}^3$

h. $\pi \int_0^2 [(36 - (2 + x^2)^2] dx = \frac{704\pi}{15} \text{ un}^3$