REVIEW EXERCISES FOR CHAPTER 3

1. *Think About It* Give the definition of a critical number, and graph an arbitrary function f showing the different types of critical numbers.

2. Consider the odd function f that is continuous, is differentiable, and has the functional values shown in the table.

x	-5	-4	-1	0	2	3	6
$f(x)$	1	3	2	0	-1	-4	0

(a) Determine $f(4)$.

(b) Determine $f(-3)$.

(c) Plot the points and make a possible sketch of the graph of f on the interval $[-6, 6]$. What is the smallest number of critical points in the interval? Explain.

(d) Does there exist at least one real number c in the interval $(-6, 6)$ where $f'(c) = -1$? Explain.

(e) Is it possible that $\lim_{x \to 0} f(x)$ does not exist? Explain.

(f) Is it necessary that $f'(x)$ exists at $x = 2$? Explain.

In Exercises 3 and 4, locate the absolute extrema of the function on the closed interval. Use a graphing utility to graph the function over the indicated interval to confirm your results.

3. $g(x) = 2x + 5\cos x, \quad [0, 2\pi]$

4. $f(x) = \dfrac{x}{\sqrt{x^2+1}}, \quad [0, 2]$

5. *Think About It* Consider the function $f(x) = 3 - |x - 4|$.

(a) Graph the function and verify that $f(1) = f(7)$.

(b) Note that $f'(x)$ is not equal to zero for any x in $[1, 7]$. Explain why this does not contradict Rolle's Theorem.

6. *Think About It* Can the Mean Value Theorem be applied to the function $f(x) = 1/x^2$ on the interval $[-2, 1]$? Explain.

In Exercises 7–10, find the point(s) guaranteed by the Mean Value Theorem for the indicated interval.

7. $f(x) = x^{2/3}$
$1 \le x \le 8$

8. $f(x) = \dfrac{1}{x}$
$1 \le x \le 4$

9. $f(x) = x - \cos x$
$-\dfrac{\pi}{2} \le x \le \dfrac{\pi}{2}$

10. $f(x) = \sqrt{x} - 2x$
$0 \le x \le 4$

11. For the function $f(x) = Ax^2 + Bx + C$, determine the value of c guaranteed by the Mean Value Theorem on the interval $[x_1, x_2]$.

12. Demonstrate the result of Exercise 11 for $f(x) = 2x^2 - 3x + 1$ on the interval $[0, 4]$.

In Exercises 13–16, find the critical numbers (if any) and the open intervals on which the function is increasing or decreasing.

13. $f(x) = (x-1)^2(x-3)$

14. $g(x) = (x+1)^3$

15. $h(x) = \sqrt{x}(x-3)$

16. $f(x) = \sin x + \cos x, \quad 0 \le x \le 2\pi$

In Exercises 17 and 18, use the First Derivative Test to find any relative extrema of the function. Use a graphing utility to verify your results.

17. $h(t) = \dfrac{1}{4}t^4 - 8t$

18. $g(x) = \dfrac{3}{2}\sin\left(\dfrac{\pi x}{2} - 1\right), \quad [0, 4]$

In Exercises 19 and 20, find any points of inflection of the function.

19. $f(x) = x + \cos x$
$0 \le x \le 2\pi$

20. $f(x) = (x+2)^2(x-4)$

In Exercises 21–24, find the limit.

21. $\lim_{x \to \infty} \dfrac{2x^2}{3x^2 + 5}$

22. $\lim_{x \to \infty} \dfrac{2x}{3x^2 + 5}$

23. $\lim_{x \to \infty} \dfrac{5\cos x}{x}$

24. $\lim_{x \to \infty} \dfrac{3x}{\sqrt{x^2 + 4}}$

In Exercises 25–28, find any vertical and horizontal asymptotes of the graph of the function. Use a graphing utility to verify your results.

25. $h(x) = \dfrac{2x+3}{x-4}$

26. $g(x) = \dfrac{5x^2}{x^2+2}$

27. $f(x) = \dfrac{3}{x} - 2$

28. $f(x) = \dfrac{3x}{\sqrt{x^2+2}}$

In Exercises 29–32, use a graphing utility to graph the function. Use the graph to approximate any relative extrema or asymptotes.

29. $f(x) = x^3 + \dfrac{243}{x}$

30. $f(x) = |x^3 - 3x^2 + 2x|$

31. $f(x) = \dfrac{x-1}{1+3x^2}$

32. $g(x) = \dfrac{\pi^2}{3} - 4\cos x + \cos 2x$

In Exercises 33–50, make use of domain, range, symmetry, asymptotes, intercepts, relative extrema, and/or points of inflection to obtain an accurate graph of the function.

33. $f(x) = 4x - x^2$

34. $f(x) = 4x^3 - x^4$

35. $f(x) = x\sqrt{16 - x^2}$

36. $f(x) = (x^2 - 4)^2$

37. $f(x) = (x - 1)^3(x - 3)^2$

38. $f(x) = (x - 3)(x + 2)^3$

39. $f(x) = x^{1/3}(x + 3)^{2/3}$

40. $f(x) = (x - 2)^{1/3}(x + 1)^{2/3}$

41. $f(x) = \dfrac{x + 1}{x - 1}$

42. $f(x) = \dfrac{2x}{1 + x^2}$

43. $f(x) = \dfrac{4}{1 + x^2}$

44. $f(x) = \dfrac{x^2}{1 + x^4}$

45. $f(x) = x^3 + x + \dfrac{4}{x}$

46. $f(x) = x^2 + \dfrac{1}{x}$

47. $f(x) = |x^2 - 9|$

48. $f(x) = |x - 1| + |x - 3|$

49. $f(x) = x + \cos x, \quad 0 \le x \le 2\pi$

50. $f(x) = \dfrac{1}{\pi}(2 \sin \pi x - \sin 2\pi x), \quad -1 \le x \le 1$

51. ***Minimum Distance*** At noon, ship A is 100 kilometers due east of ship B. Ship A is sailing west at 12 kilometers per hour, and ship B is sailing south at 10 kilometers per hour. At what time will the ships be nearest to each other, and what will this distance be?

52. ***Maximum Area*** Find the dimensions of the rectangle of maximum area, with sides parallel to the coordinate axes, that can be inscribed in the ellipse given by

$$\frac{x^2}{144} + \frac{y^2}{16} = 1.$$

53. ***Minimum Length*** A right triangle in the first quadrant has the coordinate axes as sides, and the hypotenuse passes through the point $(1, 8)$. Find the vertices of the triangle such that the length of the hypotenuse is minimum.

54. ***Minimum Length*** The wall of a building is to be braced by a beam that must pass over a parallel fence 5 feet high and 4 feet from the building. Find the length of the shortest beam that can be used.

55. ***Maximum Area*** Three sides of a trapezoid have the same length s. Of all such possible trapezoids, show that the one of maximum area has a fourth side of length $2s$.

56. ***Maximum Area*** Show that the greatest area of any rectangle inscribed in a triangle is one-half that of the triangle.

57. ***Minimum Distance*** Find the length of the longest pipe that can be carried level around a right-angle corner at the intersection of two corridors of widths 4 feet and 6 feet. (Do not use trigonometry.)

58. ***Minimum Distance*** Rework Exercise 57, given corridors of widths a meters and b meters.

59. ***Minimum Distance*** A hallway of width 6 feet meets a hallway of width 9 feet at right angles. Find the length of the longest pipe that can be carried level around this corner. [Hint: If L is the length of the pipe, show that

$$L = 6 \csc \theta + 9 \csc \left(\frac{\pi}{2} - \theta\right)$$

where θ is the angle between the pipe and the wall of the narrower hallway.]

60. ***Minimum Distance*** Rework Exercise 59, given that one hallway is of width a meters and the other is of width b meters. Show that the result is the same as in Exercise 58.

61. ***Harmonic Motion*** The height of an object attached to a spring is given by the harmonic equation

$$y = \tfrac{1}{3} \cos 12t - \tfrac{1}{4} \sin 12t$$

where y is measured in inches and t is measured in seconds.

(a) Calculate the height and velocity of the object when $t = \pi/8$ second.

(b) Show that the maximum displacement of the object is $\frac{5}{12}$ inch.

(c) Find the period P of y. Also, find the frequency f (number of oscillations per second) if $f = 1/P$.

62. ***Writing*** The general equation giving the height of an oscillating object attached to a spring is

$$y = A \sin \sqrt{\frac{k}{m}}\, t + B \cos \sqrt{\frac{k}{m}}\, t$$

where k is the spring constant and m is the mass of the object.

(a) Show that the maximum displacement of the object is $\sqrt{A^2 + B^2}$.

(b) Show that the object oscillates with a frequency of

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}.$$

(c) How is the frequency changed if the stiffness k of the spring is increased?

(d) How is the frequency changed if the mass m of the object is increased?

In Exercises 63 and 64, use Newton's Method to approximate any real zeros of the function accurate to three decimal places. Use the root-finding capabilities of a graphing utility to verify your results.

63. $f(x) = x^3 - 3x - 1$

64. $f(x) = x^3 + 2x + 1$

In Exercises 65 and 66, use Newton's Method to approximate, to three decimal places, the x-value of the points of intersection of the equations. Use a graphing utility to verify your results.

65. $y = x^4$
$y = x + 3$

66. $y = \sin \pi x$
$y = 1 - x$