CHAPTER 6
Applications of Integration

PTER 6

ations of Integration

6.1 Area of a Region Between Two Curves

$[0 - (x^2 - 6x)]\,dx = -\int_0^6 (x^2 - 6x)\,dx$

$[(2x + 5) - (x^2 + 2x + 1)]\,dx = \int_{-2}^{2} (-x^2 + 4)\,dx$

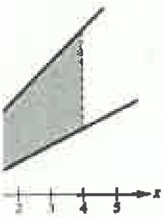
$[(-x^2 + 2x + 3) - (x^2 - 4x + 3)]\,dx = \int_0^3 (-2x^2 + 6x)\,dx$

$(x^2 - x^3)\,dx$

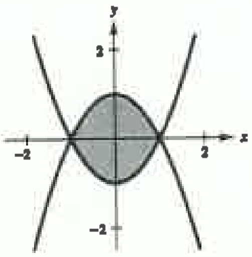
$\int_{-1}^{0} 3(x^3 - x)\,dx = 6\int_{-1}^{0} (x^3 - x)\,dx \quad \text{or} \quad -6\int_0^1 (x^3 - x)\,dx$

$\int_0^1 [(x - 1)^3 - (x - 1)]\,dx$

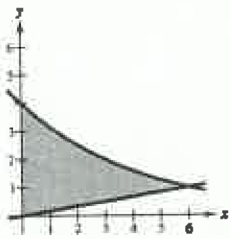
$+ 1) - \frac{x}{2}\Big]\,dx$



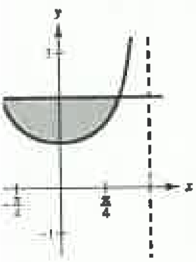
8. $\int_{-1}^{1} [(1 - x^2) - (x^2 - 1)]\,dx$



9. $\int_0^6 \left[4(2^{-x/3}) - \frac{x}{6}\right] dx$



$2 - \sec x]\,dx$

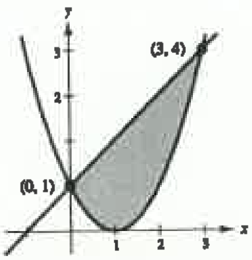


11. $f(x) = x + 1$

$g(x) = (x - 1)^2$

$A \approx 4$

Matches (d)

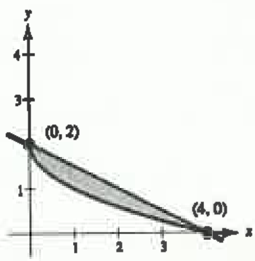


12. $f(x) = 2 - \frac{1}{2}x$

$g(x) = 2 - \sqrt{x}$

$A \approx 1$

Matches (a)



13. The points of intersection are given by:

$$x^2 - 4x = 0$$

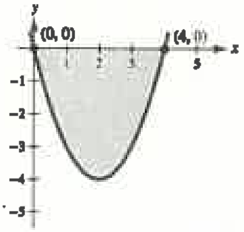
$$x(x-4) = 0 \quad \text{when} \quad x = 0, 4$$

$$A = \int_0^4 [g(x) - f(x)]\,dx$$

$$= -\int_0^4 (x^2 - 4x)\,dx$$

$$= -\left[\frac{x^3}{3} - 2x^2\right]_0^4$$

$$= \frac{32}{3}$$



14. The points of intersection are given by:

$$3 - 2x - x^2 = 0$$

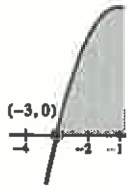
$$(3 + x)(1 - x) = 0 \quad \text{when} \quad x = -3, 1$$

$$A = \int_{-3}^1 [f(x) - g(x)]\,dx$$

$$= \int_{-3}^1 (3 - 2x - x^2)\,dx$$

$$= \left[3x - x^2 - \frac{x^3}{3}\right]_{-3}^1$$

$$= \frac{32}{3}$$



15. The points of intersection are given by:

$$x^2 + 2x + 1 = 3x + 3$$

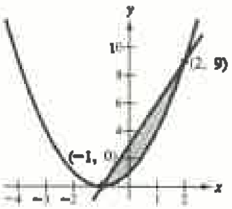
$$(x-2)(x+1) = 0 \quad \text{when} \quad x = -1, 2$$

$$A = \int_{-1}^2 [g(x) - f(x)]\,dx$$

$$= \int_{-1}^2 [(3x + 3) - (x^2 + 2x + 1)]\,dx$$

$$= \int_{-1}^2 (2 + x - x^2)\,dx$$

$$= \left[2x + \frac{x^2}{2} - \frac{x^3}{3}\right]_{-1}^2 = \frac{9}{2}$$



16. The points of intersection are given by:

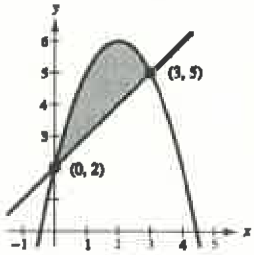
$$-x^2 + 4x + 2 = x + 2$$

$$x(3 - x) = 0 \quad \text{when} \quad x = 0, 3$$

$$A = \int_0^3 [f(x) - g(x)]\,dx$$

$$= \int_0^3 [(-x^2 + 4x + 2) - (x + 2)]\,dx$$

$$= \int_0^3 (-x^2 + 3x)\,dx = \left[\frac{-x^3}{3} + \frac{3}{2}x^2\right]_0^3 =$$



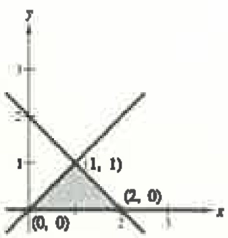
17. The points of intersection are given by:

$$x = 2 - x \quad \text{and} \quad x = 0 \quad \text{and} \quad 2 - x = 0$$

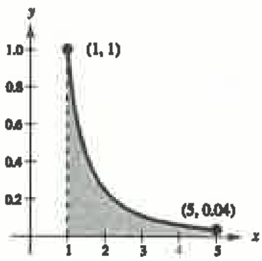
$$x = 1 \qquad\qquad x = 0 \qquad\qquad x = 2$$

$$A = \int_0^1 [(2 - y) - (y)]\,dy = \left[2y - y^2\right]_0^1 = 1$$

Note that if we integrate with respect to x, we need two integrals. Also, note that the region is a triangle.



18. $A = \displaystyle\int_1^5 \left(\frac{1}{x^2} - 0\right) dx = \left[-\frac{1}{x}\right]_1^5 = \frac{4}{5}$



nts of intersection are given by:

$\sqrt{3x} + 1 = x + 1$

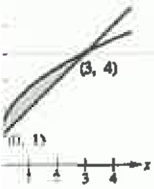
$\sqrt{3x} = x$ when $x = 0, 3$

$[f(x) - g(x)]\,dx$

$\left[\left(\sqrt{3x} + 1\right) - (x + 1)\right] dx$

$[(3x)^{1/2} - x]\,dx$

$(3x)^{3/2} - \left.\frac{x^2}{2}\right]_0^3 = \frac{3}{2}$



20. The points of intersection are given by:

$$\sqrt[3]{x} = x$$

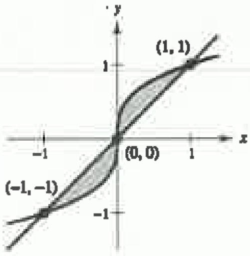
$$x = -1, 0, 1$$

$$A = 2\int_0^1 [f(x) - g(x)]\,dx$$

$$= 2\int_0^1 \left(\sqrt[3]{x} - x\right) dx$$

$$= 2\int_0^1 (x^{1/3} - x)\,dx$$

$$= 2\left[\frac{3}{4}x^{4/3} - \frac{1}{2}x^2\right]_0^1 = \frac{1}{2}$$



nts of intersection are given by:

$y^2 = y + 2$

$(y - 2)(y + 1) = 0$ when $y = -1, 2$

$[g(y) - f(y)]\,dy$

$[(y + 2) - y^2]\,dy$

$y + \frac{y^2}{2} - \left.\frac{y^3}{3}\right]_{-1}^{2} = \frac{9}{2}$

22. The points of intersection are given by:

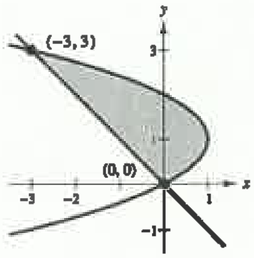
$$2y - y^2 = -y$$

$$y(y - 3) = 0 \text{ when } y = 0, 3$$

$$A = \int_0^3 [f(y) - g(y)]\,dy$$

$$= \int_0^3 [(2y - y^2) - (-y)]\,dy$$

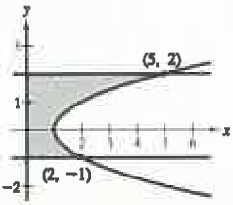
$$= \int_0^3 (3y - y^2)\,dy = \left[\frac{3}{2}y^2 - \frac{1}{3}y^3\right]_0^3 = \frac{9}{2}$$



$[f(y) - g(y)]\,dy$

$[(y^2 + 1) - 0]\,dy$

$\frac{y^3}{3} + y\Big]_{-1}^{2} = 6$

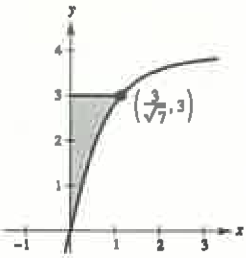


24. $A = \int_0^3 [f(y) - g(y)]\, dy$

$= \int_0^3 \left[\frac{y}{\sqrt{16 - y^2}} - 0\right] dy$

$= -\frac{1}{2}\int_0^3 (16 - y^2)^{-1/2}(-2y)\, dy$

$= \left[-\sqrt{16 - y^2}\right]_0^3 = 4 - \sqrt{7} \approx 1.354$



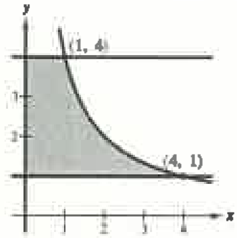
25. $y = \frac{4}{x} \Rightarrow x = \frac{4}{y}$

$A = \int_1^4 \left[\frac{4}{y} - 0\right] dy$

$= \left[4 \ln|y|\right]_1^4$

$= 4 \ln 4$

$= 8 \ln 2 \approx 5.545$

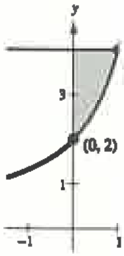


26. $A = \int_0^1 \left(4 - \frac{4}{2 - x}\right) dx$

$= \left[4x + 4 \ln|2 - x|\right]_0^1$

$= 4 - 4 \ln 2$

≈ 1.227



27. The points of intersection are given by:

$x^3 - 3x^2 + 3x = x^2$

$x(x - 1)(x - 3) = 0$ when $x = 0, 1, 3$

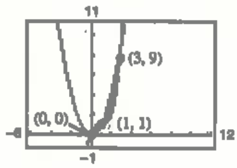
$A = \int_0^1 [f(x) - g(x)]\, dx + \int_1^3 [g(x) - f(x)]\, dx$

$= \int_0^1 [(x^3 - 3x^2 + 3x) - x^2]\, dx + \int_1^3 [x^2 - (x^3 - 3x^2 + 3x)]\, dx$

$= \int_0^1 (x^3 - 4x^2 + 3x)\, dx + \int_1^3 (-x^3 + 4x^2 - 3x)\, dx$

$= \left[\frac{x^4}{4} - \frac{4}{3}x^3 + \frac{3}{2}x^2\right]_0^1 + \left[\frac{-x^4}{4} + \frac{4}{3}x^3 - \frac{3}{2}x^2\right]_1^3 = \frac{37}{12}$

Numerical Approximation: $0.417 + 2.667 \approx 3.083$



28. The point of intersection is given by:

$x^3 - 2x + 1 = -2x$

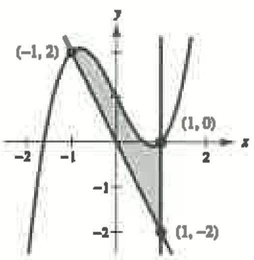
$x^3 + 1 = 0$ when $x = -1$

$A = \int_{-1}^1 [f(x) - g(x)]\, dx$

$= \int_{-1}^1 [(x^3 - 2x + 1) - (-2x)]\, dx$

$= \int_{-1}^1 (x^3 + 1)\, dx = \left[\frac{x^4}{4} + x\right]_{-1}^1 = 2$

Numerical Approximation: 2.0



its of intersection are given by:

$-4x + 3 = 3 + 4x - x^2$

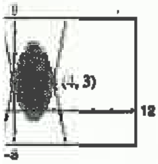
$x(x - 4) = 0$ when $x = 0, 4$

$[(3 + 4x - x^2) - (x^2 - 4x + 3)]\,dx$

$(-2x^2 + 8x)\,dx$

$\frac{2x^3}{3} + 4x^2\Big]_0^4 = \frac{64}{3}$

al Approximation: 21.333



30. The points of intersection are given by:

$$x^4 - 2x^2 = 2x^2$$

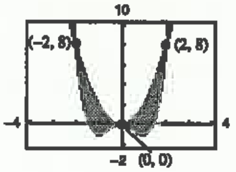
$$x^2(x^2 - 4) = 0 \quad \text{when} \quad x = 0, \pm 2$$

$$A = 2\int_0^2 [2x^2 - (x^4 - 2x^2)]\,dx$$

$$= 2\int_0^2 (4x^2 - x^4)\,dx$$

$$= 2\left[\frac{4x^3}{3} - \frac{x^5}{5}\right]_0^2 = \frac{128}{15}$$

Numerical Approximation: 8.533



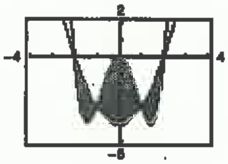
$^4 - 4x^2$, $g(x) = x^2 - 4$

its of intersection are given by:

$x^4 - 4x^2 = x^2 - 4$

$^4 - 5x^2 + 4 = 0$

$-4)(x^2 - 1) = 0$ when $x = \pm 2, \pm 1$



netry,

$$2\int_0^1 [(x^4 - 4x^2) - (x^2 - 4)]\,dx + 2\int_1^2 [(x^2 - 4) - (x^4 - 4x^2)]\,dx$$

$$2\int_0^1 (x^4 - 5x^2 + 4)\,dx + 2\int_1^2 (-x^4 + 5x^2 - 4)\,dx$$

$$2\left[\frac{x^5}{5} - \frac{5x^3}{3} + 4x\right]_0^1 + 2\left[-\frac{x^5}{5} + \frac{5x^3}{3} - 4x\right]_1^2$$

$$2\left[\frac{1}{5} - \frac{5}{3} + 4\right] + 2\left[\left(-\frac{32}{5} + \frac{40}{3} - 8\right) - \left(-\frac{1}{5} + \frac{5}{3} - 4\right)\right] = 8.$$

al Approximation: $5.067 + 2.933 = 8.0$

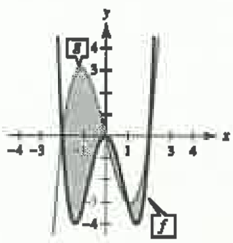
$^4 - 4x^2$, $g(x) = x^3 - 4x$

its of intersection are given by:

$$x^4 - 4x^2 = x^3 - 4x$$

$^4 - x^3 - 4x^2 + 4x = 0$

$-1)(x + 2)(x - 2) = 0$ when $x = -2, 0, 1, 2$



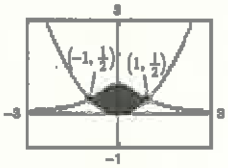
$$[(x^3 - 4x) - (x^4 - 4x^2)]dx + \int_0^1 [(x^4 - 4x^2) - (x^3 - 4x)]dx + \int_1^2 [(x^3 - 4x) - (x^4 - 4x^2)]dx$$

$$\frac{1}{} + \frac{37}{60} + \frac{53}{60} = \frac{293}{30}$$

al Approximation: $8.267 + 0.617 + 0.883 \approx 9.767$

33. The points of intersection are given by:

$$\frac{1}{1+x^2} = \frac{x^2}{2}$$
$$x^4 + x^2 - 2 = 0$$
$$(x^2+2)(x^2-1) = 0$$
$$x = \pm 1$$



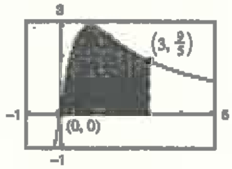
$$A = 2\int_0^1 [f(x) - g(x)]\,dx$$
$$= 2\int_0^1 \left[\frac{1}{1+x^2} - \frac{x^2}{2}\right]dx$$
$$= 2\left[\arctan x - \frac{x^3}{6}\right]_0^1$$
$$= 2\left(\frac{\pi}{4} - \frac{1}{6}\right) = \frac{\pi}{2} - \frac{1}{3} \approx 1.237$$

Numerical Approximation: 1.237

34. $A = \int_0^3 \left[\frac{6x}{x^2+1} - 0\right]dx$

$$= \left[3\ln(x^2+1)\right]_0^3$$
$$= 3\ln 10$$
$$\approx 6.908$$

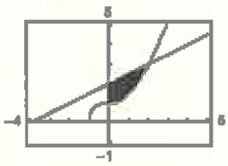
Numerical Approximation: 6.908



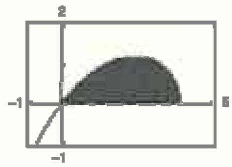
35. $\sqrt{1+x^3} \le \frac{1}{2}x + 2$ on $[0, 2]$

Numerical approximation: 1.759

$$A = \int_0^2 \left[\frac{1}{2}x + 2 - \sqrt{1+x^3}\right]dx \approx 1.759$$

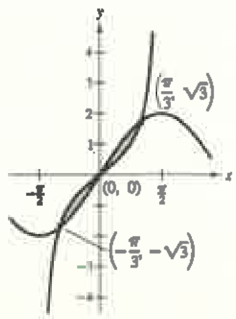


36. $A = \int_0^4 x\sqrt{\frac{4-x}{4+x}}\,dx \approx 3.434$



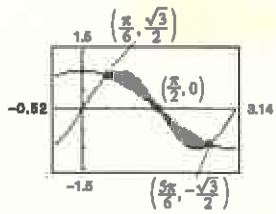
37. $A = 2\int_0^{\pi/3} [f(x) - g(x)]\,dx$

$$= 2\int_0^{\pi/3} (2\sin x - \tan x)\,dx$$
$$= 2\Big[-2\cos x + \ln|\cos x|\Big]_0^{\pi/3}$$
$$= 2(1 - \ln 2) \approx 0.614$$



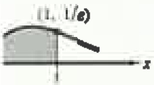
38. $A = 2\int_{\pi/6}^{\pi/2} [f(x) - g(x)]\,dx$

$$= 2\int_{\pi/6}^{\pi/2} [\sin 2x - \cos x]\,dx$$
$$= 2\left[-\frac{1}{2}\cos 2x - \sin x\right]_{\pi/6}^{\pi/2}$$
$$= \frac{1}{2}$$



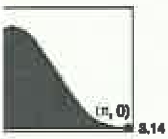
$[xe^{-x^2} - 0]\,dx$

$\frac{1}{2}e^{-x^2}\Big]_0^1 = \frac{1}{2}\left(1 - \frac{1}{e}\right) \approx 0.316$



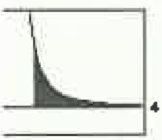
$[(2\sin x + \sin 2x) - 0]\,dx$

$-2\cos x - \frac{1}{2}\cos 2x\Big]_0^{\pi} = 4.0$



$\left[\frac{1}{x^2}e^{1/x} - 0\right]dx$

$-e^{1/x}\Big]_1^3 = e - e^{1/3} \approx 1.323$



$\sqrt{\frac{x^3}{4-x}},\quad y = 0,\quad x = 3$

$\int_0^3 \sqrt{\frac{x^3}{4-x}}\,dx,$

, it cannot be evaluated by hand.

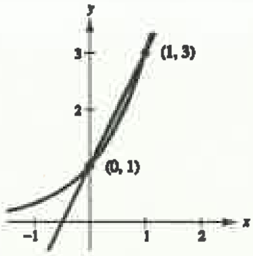
$\sqrt{x}\,e^x,\ y = 0,\ x = 0,\ x = 1$

$\int_0^1 \sqrt{x}\,e^x\,dx.$

, it cannot be evaluated by hand.

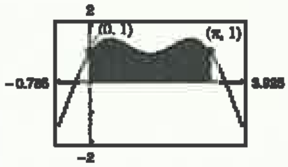
40. From the graph we see that f and g intersect twice at $x = 0$ and $x = 1$.

$$\begin{aligned} A &= \int_0^1 [g(x) - f(x)]\,dx \\ &= \int_0^1 [(2x+1) - 3^x]\,dx \\ &= \left[x^2 + x - \frac{1}{\ln 3}(3^x)\right]_0^1 \\ &= 2\left(1 - \frac{1}{\ln 3}\right) \approx 0.180 \end{aligned}$$



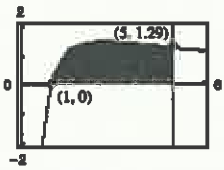
42. $A = \int_0^{\pi} [(2\sin x + \cos 2x) - 0]\,dx$

$= \left[-2\cos x + \frac{1}{2}\sin 2x\right]_0^{\pi} = 4$

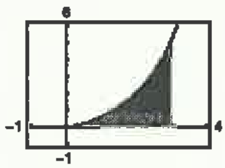


44. $A = \int_1^5 \left[\frac{4\ln x}{x} - 0\right]dx$

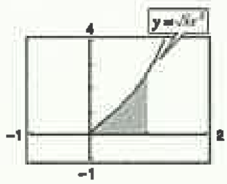
$= \left[2(\ln x)^2\right]_1^5 = 2(\ln 5)^2 \approx 5.181$



(c) 4.7721



(c) 1.2556

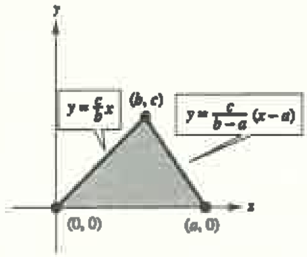


47. $A = \int_0^c \left[\left(\frac{b-a}{c}y + a\right) - \frac{b}{c}y\right] dy$

$= \int_0^c \left(-\frac{a}{c}y + a\right) dy$

$= \left[-\frac{a}{2c}y^2 + ay\right]_0^c$

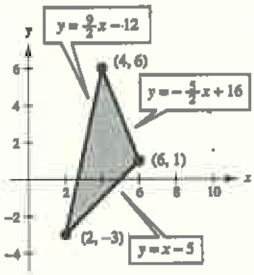
$= -\frac{ac}{2} + ac = \frac{ac}{2} \quad \left(= \frac{1}{2}\text{(base)(height)}\right)$



48. $A = \int_2^4 \left[\left(\frac{9}{2}x - 12\right) - (x-5)\right] dx + \int_4^6 \left[\left(-\frac{5}{2}x + 16\right) - (x-5)\right] dx$

$= \int_2^4 \left(\frac{7}{2}x - 7\right) dx + \int_4^6 \left(-\frac{7}{2}x + 21\right) dx$

$= \left[\frac{7}{4}x^2 - 7x\right]_2^4 + \left[-\frac{7}{4}x^2 + 21x\right]_4^6 = 7 + 7 = 14$



49. $f(x) = x^3$

$f'(x) = 3x^2$

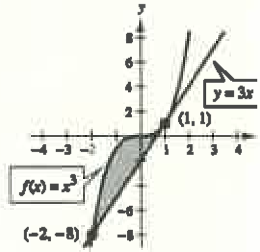
At $(1, 1)$, $f'(1) = 3$.

Tangent line:

$y - 1 = 3(x - 1)$ or $y = 3x - 2$

The tangent line intersects $f(x) = x^3$ at $x = -2$.

$$A = \int_{-2}^{1} [x^3 - (3x - 2)]\, dx = \left[\frac{x^4}{4} - \frac{3x^2}{2} + 2x\right]_{-2}^{1} = \frac{27}{4}$$



50. $f(x) = \frac{1}{x^2 + 1}$

$f'(x) = -\frac{2x}{(x^2 + 1)^2}$

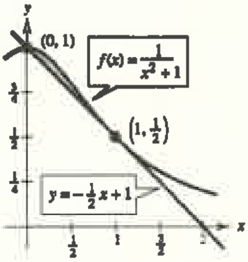
At $\left(1, \frac{1}{2}\right)$, $f'(1) = -\frac{1}{2}$.

Tangent line:

$y - \frac{1}{2} = -\frac{1}{2}(x - 1)$ or $y = -\frac{1}{2}x + 1$

The tangent line intersects $f(x) = \frac{1}{x^2 + 1}$ at $x = 0$.

$$A = \int_0^1 \left[\frac{1}{x^2+1} - \left(-\frac{1}{2}x + 1\right)\right] dx = \left[\arctan x + \frac{x^2}{4} - x\right]_0^1 = \frac{\pi - 3}{4} \approx 0.0354$$



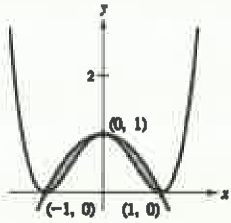
$^2 + 1 \le 1 - x^2$ on $[-1, 1]$

$[(1 - x^2) - (x^4 - 2x^2 + 1)]\,dx$

$(x^2 - x^4)\,dx$

$- \frac{x^5}{5}\Big]_{-1}^{1} = \frac{4}{15}$

use a single integral because $x^4 - 2x^2 + 1 \le 1 - x^2$ on $[-1, 1]$.



on $[-1, 0]$

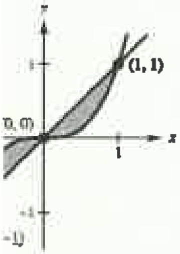
on $[0, 1]$

ctions symmetric to origin

$(x^3 - x)\,dx = -\int_0^1 (x^3 - x)\,dx.$

$(x^3 - x)\,dx = 0.$

$2\int_0^1 (x - x^3)\,dx = 2\left[\frac{x^2}{2} - \frac{x^4}{4}\right]_0^1 = \frac{1}{2}$



53. $$A = \int_{-3}^{3} (9 - x^2)\,dx = 36$$

$$\int_{-\sqrt{9-b}}^{\sqrt{9-b}} [(9 - x^2) - b]\,dx = 18$$

$$\int_0^{\sqrt{9-b}} [(9 - b) - x^2]\,dx = 9$$

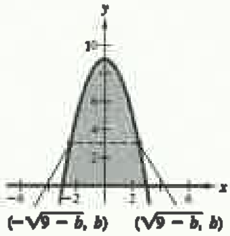
$$\left[(9 - b)x - \frac{x^3}{3}\right]_0^{\sqrt{9-b}} = 9$$

$$\frac{2}{3}(9 - b)^{3/2} = 9$$

$$(9 - b)^{3/2} = \frac{27}{2}$$

$$9 - b = \frac{9}{\sqrt[3]{4}}$$

$$b = 9 - \frac{9}{\sqrt[3]{4}} \approx 3.330$$



$$(9 - x)\,dx = 2\left[9x - \frac{x^2}{2}\right]_0^9 = 81$$

$$2\int_0^{9-b} [(9 - x) - b]\,dx = \frac{81}{2}$$

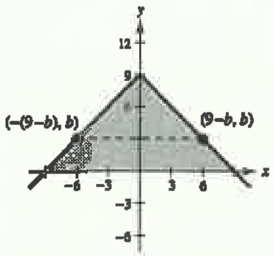
$$2\int_0^{9-b} [(9 - b) - x]\,dx = \frac{81}{2}$$

$$2\left[(9 - b)x - \frac{x^2}{2}\right]_0^{9-b} = \frac{81}{2}$$

$$(9 - b)(9 - b) = \frac{81}{2}$$

$$9 - b = \frac{9}{\sqrt{2}}$$

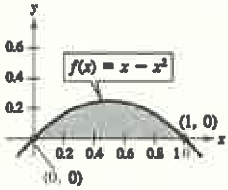
$$b = 9 - \frac{9}{\sqrt{2}} \approx 2.636$$



55. $\lim_{\|\Delta\| \to 0} \sum_{i=1}^{n} (x_i - x_i^2)\,\Delta x$

where $x_i = \frac{i}{n}$ and $\Delta x = \frac{1}{n}$ is the same as

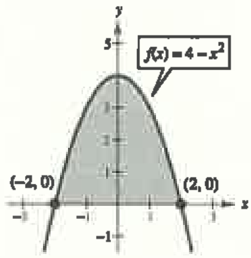
$$\int_0^1 (x - x^2)\,dx = \left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1 = \frac{1}{6}.$$



56. $\lim_{\|\Delta\| \to 0} \sum_{i=1}^{n} (4 - x_i^2)\,\Delta x$

where $x_i = -2 + \frac{4i}{n}$ and $\Delta x = \frac{4}{n}$ is the sam

$$\int_{-2}^{2} (4 - x^2)\,dx = \left[4x - \frac{x^3}{3}\right]_{-2}^{2} = \frac{32}{3}$$



57. $\int_0^5 [(7.21 + 0.58t) - (7.21 + 0.45t)]\,dt = \int_0^5 0.13t\,dt = \left[\frac{0.13t^2}{2}\right]_0^5 = \$1.625$ billion

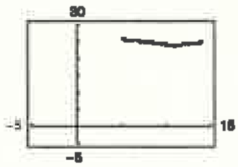
58. $\int_0^5 [(7.21 + 0.26t + 0.02t^2) - (7.21 + 0.1t + 0.01t^2)]\,dt = \int_0^5 (0.01t^2 + 0.16t)\,dt$

$$= \left[\frac{0.01t^3}{3} + \frac{0.16t^2}{2}\right]_0^5$$

$$= \frac{29}{12} \text{ billion} \approx \$2.417 \text{ billion}$$

59. $f(t) = \begin{cases} 27.77 - 0.36t & 5 \le t \le 10 \\ 21.00 + 0.27t & 10 \le t \le 14 \end{cases}$

(a)



(b) $\int_{10}^{14} [(21.00 + 0.27t) - (27.77 - 0.36t)]\,dt = \int_{10}^{14} [-6.77 + 0.63t]\,dt$

$$= \left[-6.77t + 0.315t^2\right]_{10}^{14} = 3.16 \text{ billion pounds}$$

60. $\int_0^{10} [(568.50 + 7.15t) - (525.60 + 6.43t)]\,dt = \int_0^{10} (42.90 + 0.72t)\,dt = \left[42.90t + 0.36t^2\right]_0^{10} = \$465$ million

61. 5% : $P_1 = 893{,}000\,e^{(0.05)t}$

$3\frac{1}{2}$%: $P_2 = 893{,}000\,e^{(0.035)t}$

Difference in profits over 5 years:

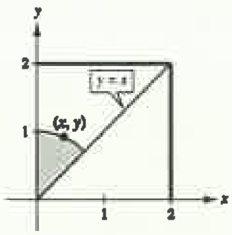
$$\int_0^5 [893{,}000e^{0.05t} - 893{,}000e^{0.035t}]\,dt = 893{,}000\left[\frac{e^{0.05t}}{0.05} - \frac{e^{0.035t}}{0.035}\right]_0^5$$

$$\approx 893{,}000[(25.6805 - 34.0356) - (20 - 28.5714)]$$

$$\approx 893{,}000(0.2163) \approx \$193{,}156$$

l 2 is better, since the cummulative deficit (the area under the curve) is less.

l area is 8 times the area of the shaded region to the right. A point (x, y) is on the upper y of the region if



$$^2 + y^2 = 2 - y$$

$$^2 + y^2 = 4 - 4y + y^2$$

$$x^2 = 4 - 4y$$

$$4y = 4 - x^2$$

$$y = 1 - \frac{x^2}{4}.$$

determine where this curve intersects the line $y = x$.

$$x = 1 - \frac{x^2}{4}$$

$$+ 4x - 4 = 0$$

$$x = \frac{-4 \pm \sqrt{16 + 16}}{2} = -2 \pm 2\sqrt{2} \implies x = -2 + 2\sqrt{2}$$

$$\text{ea} = 8\int_0^{-2+2\sqrt{2}} \left(1 - \frac{x^2}{4} - x\right) dx$$

$$= 8\left[x - \frac{x^3}{12} - \frac{x^2}{2}\right]_0^{-2+2\sqrt{2}} \approx 8(0.4379) = 3.503$$

ves intersect at the point where the slope of y_2 equals that of y_1, 1.

$$8x^2 + k \implies y'_2 = 0.16x = 1 \implies x = \frac{1}{.16} = 6.25$$

value of k is given by

$$y_1 = y_2$$

$$6.25 = (0.08)(6.25)^2 + k$$

$$k = 3.125.$$

(b) $\text{Area} = 2\int_0^{6.25} (y_2 - y_1)\, dx$

$$= 2\int_0^{6.25} (0.08x^2 + 3.125 - x)\, dx$$

$$= 2\left[\frac{0.08x^3}{3} + 3.125x - \frac{x^2}{2}\right]_0^{6.25}$$

$$= 2(6.510417) \approx 13.02083$$

$$2\left[\int_0^5 \left(1 - \frac{1}{3}\sqrt{5 - x}\right) dx + \int_5^{5.5} (1 - 0)\, dx\right]$$

$$2\left(\left[x + \frac{2}{9}(5 - x)^{3/2}\right]_0^5 + \left[x\right]_5^{5.5}\right) = 2\left(5 - \frac{10\sqrt{5}}{9} + 5.5 - 5\right) \approx 6.031 \text{ m}^2$$

$2A \approx 2(6.031) \approx 12.062$ m^3

(c) $5000\, V \approx 5000(12.062) = 60{,}310$ pounds

$$6.031 - 2\left[\pi\left(\frac{1}{16}\right)^2\right] - 2\left[\pi\left(\frac{1}{8}\right)^2\right] \approx 5.908$$

$2A \approx 2(5.908) \approx 11.816$ m^3

(c) $5000V \approx 5000(11.816) = 59{,}082$ pounds

67. $50 - 0.5x = 0.125x$

$$x = 80$$

$p_1(80) = p_2(80) = 10$
Point of equilibrium: $(80, 10)$

$$CS = \int_0^{80} [(50 - 0.5x) - 10]\,dx$$

$$= \left[-\frac{0.5x^2}{2} + 40x\right]_0^{80} = 1600$$

$$PS = \int_0^{80} [10 - 0.125x]\,dx$$

$$= \left[10x - \frac{0.125x^2}{2}\right]_0^{80} = 400$$

68. $1000 - 0.4x^2 = 42x$

$$x = 20$$

$p_1(20) = p_2(20) = 840$
Point of equilibrium: $(20, 840)$

$$CS = \int_0^{20} [(1000 - 0.4x^2) - 840]\,dx$$

$$= \left[160x - \frac{0.4x^3}{3}\right]_0^{20} \approx 2133.33$$

$$PS = \int_0^{20} [840 - 42x]\,dx$$

$$= \left[840x - 21x^2\right]_0^{20} = 8400$$

69. True

70. True

Section 6.2 Volume: The Disc Method

1. $V = \pi\int_0^1 (-x + 1)^2\,dx = \pi\int_0^1 (x^2 - 2x + 1)\,dx = \pi\left[\frac{x^3}{3} - x^2 + x\right]_0^1 = \frac{\pi}{3}$

2. $V = \pi\int_0^2 (4 - x^2)^2\,dx = \pi\int_0^2 (x^4 - 8x^2 + 16)\,dx = \pi\left[\frac{x^5}{5} - \frac{8x^3}{3} + 16x\right]_0^2 = \frac{256\pi}{15}$

3. $V = \pi\int_1^4 (\sqrt{x})^2\,dx = \pi\int_1^4 x\,dx = \pi\left[\frac{x^2}{2}\right]_1^4 = \frac{15\pi}{2}$

4. $V = \pi\int_0^2 (\sqrt{4 - x^2})^2\,dx = \pi\int_0^2 (4 - x^2)\,dx = \pi\left[4x - \frac{x^3}{3}\right]_0^2 = \frac{16\pi}{3}$

5. $V = \pi\int_0^1 [(x^2)^2 - (x^3)^2]\,dx = \pi\int_0^1 (x^4 - x^6)\,dx = \pi\left[\frac{x^5}{5} - \frac{x^7}{7}\right]_0^1 = \frac{2\pi}{35}$

6. $2 = 4 - \frac{x^2}{4}$

$8 = 16 - x^2$

$x^2 = 8$

$x = \pm 2\sqrt{2}$

$$V = \pi\int_{-2\sqrt{2}}^{2\sqrt{2}} \left[\left(4 - \frac{x^2}{4}\right)^2 - (2)^2\right] dx$$

$$= 2\pi\int_0^{2\sqrt{2}} \left[\frac{x^4}{16} - 2x^2 + 12\right] dx$$

$$= 2\pi\left[\frac{x^5}{80} - \frac{2x^3}{3} + 12x\right]_0^{2\sqrt{2}}$$

$$= 2\pi\left[\frac{128\sqrt{2}}{80} - \frac{32\sqrt{2}}{3} + 24\sqrt{2}\right]$$

$$= \frac{448\sqrt{2}}{\text{[illegible]}}\pi \approx 132.69$$

$\Rightarrow x = \sqrt{y}$

$\int_0^4 (\sqrt{y})^2\,dy = \pi\int_0^4 y\,dy$

$\left.\frac{y^2}{2}\right]_0^4 = 8\pi$

8. $y = \sqrt{16 - x^2} \Rightarrow x = \sqrt{16 - y^2}$

$$V = \pi\int_0^4 \left(\sqrt{16 - y^2}\right)^2 dy = \pi\int_0^4 (16 - y^2)\,dy$$

$$= \pi\left[16y - \frac{y^3}{3}\right]_0^4 = \frac{128\pi}{3}$$

$\Rightarrow x = y^{3/2}$

$\int_0^1 (y^{3/2})^2\,dy = \pi\int_0^1 y^3\,dy = \pi\left[\frac{y^4}{4}\right]_0^1 = \frac{\pi}{4}$

10. $V = \pi\int_1^4 (-y^2 + 4y)^2\,dy = \pi\int_1^4 (y^4 - 8y^3 + 16y^2)\,dy$

$$= \pi\left[\frac{y^5}{5} - 2y^4 + \frac{16y^3}{3}\right]_1^4$$

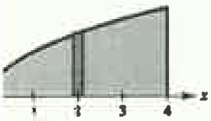
$$= \frac{459\pi}{15} = \frac{153\pi}{5}$$

$y = 0,\ x = 4$

$= \sqrt{x},\ r(x) = 0$

$\pi\int_0^4 (\sqrt{x})^2\,dx$

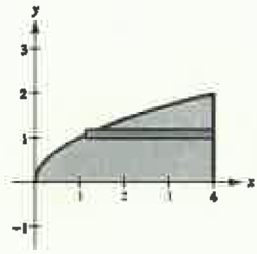
$\pi\int_0^4 x\,dx = \left[\frac{\pi}{2}x^2\right]_0^4 = 8\pi$



(b) $R(y) = 4,\ r(y) = y^2$

$$V = \pi\int_0^2 (16 - y^4)\,dy$$

$$= \pi\left[16y - \frac{1}{5}y^5\right]_0^2 = \frac{128\pi}{5}$$

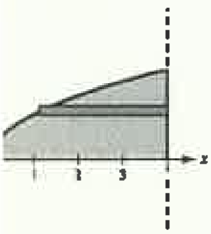


$= 4 - y^2,\ r(y) = 0$

$\pi\int_0^2 (4 - y^2)^2\,dy$

$\pi\int_0^2 (16 - 8y^2 + y^4)\,dy$

$\pi\left[16y - \frac{8}{3}y^3 + \frac{1}{5}y^5\right]_0^2 = \frac{256\pi}{15}$

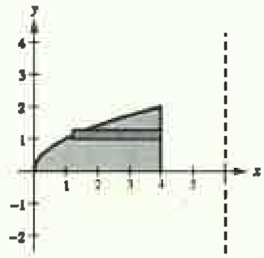


(d) $R(y) = 6 - y^2,\ r(y) = 2$

$$V = \pi\int_0^2 [(6 - y^2)^2 - 4]\,dy$$

$$= \pi\int_0^2 (32 - 12y^2 + y^4)\,dy$$

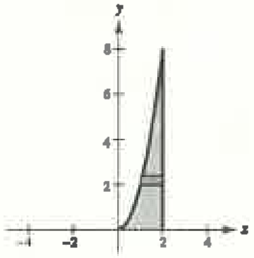
$$= \pi\left[32y - 4y^3 + \frac{1}{5}y^5\right]_0^2 = \frac{192\pi}{5}$$



12. $y = 2x^2,\ y = 0,\ x = 2$

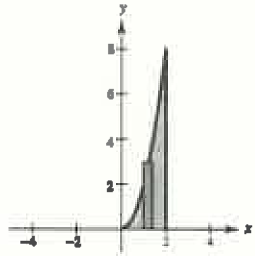
(a) $R(y) = 2,\ r(y) = \sqrt{y/2}$

$$V = \pi\int_0^8 \left(4 - \frac{y}{2}\right) dy = \pi\left[4y - \frac{y^2}{4}\right]_0^8 = 16\pi$$



(b) $R(x) = 2x^2,\ r(x) = 0$

$$V = \pi\int_0^2 4x^4\, dx = \pi\left[\frac{4x^5}{5}\right]_0^2 = \frac{128\pi}{5}$$

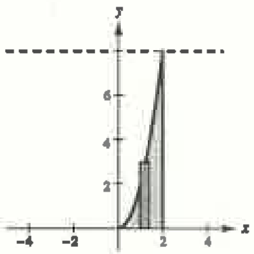


(c) $R(x) = 8,\ r(x) = 8 - 2x^2$

$$V = \pi\int_0^2 [64 - (64 - 32x^2 + 4x^4)]\, dx$$

$$= \pi\int_0^2 (32x^2 - 4x^4)\, dx = 4\pi\int_0^2 (8x^2 - x^4)\, dx$$

$$= 4\pi\left[\frac{8}{3}x^3 - \frac{1}{5}x^5\right]_0^2 = \frac{896\pi}{15}$$

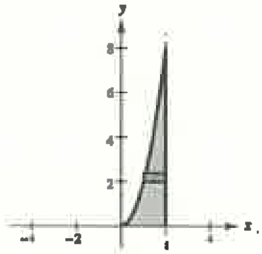


(d) $R(y) = 2 - \sqrt{y/2},\ r(y) = 0$

$$V = \pi\int_0^8 \left(2 - \sqrt{\frac{y}{2}}\right)^2 dy$$

$$= \pi\int_0^8 \left(4 - 4\sqrt{\frac{y}{2}} + \frac{y}{2}\right) dy$$

$$= \pi\left[4y - \frac{4\sqrt{2}}{3}y^{3/2} + \frac{y^2}{4}\right]_0^8 = \frac{16\pi}{3}$$



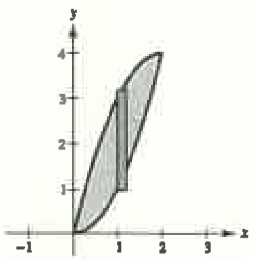
13. $y = x^2,\ y = 4x - x^2$ intersect at $(0, 0)$ and $(2, 4)$.

(a) $R(x) = 4x - x^2\ r(x) = x^2$

$$V = \pi\int_0^2 [(4x - x^2)^2 - x^4]\, dx$$

$$= \pi\int_0^2 (16x^2 - 8x^3)\, dx$$

$$= \pi\left[\frac{16}{3}x^3 - 2x^4\right]_0^2 = \frac{32\pi}{3}$$

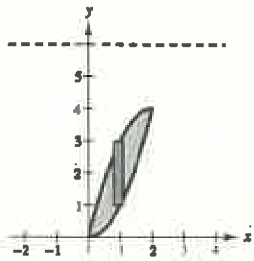


(b) $R(x) = 6 - x^2,\ r(x) = 6 - (4x - x^2)$

$$V = \pi\int_0^2 [(6 - x^2)^2 - (6 - 4x + x^2)^2]\, dx$$

$$= 8\pi\int_0^2 (x^3 - 5x^2 + 6x)\, dx$$

$$= 8\pi\left[\frac{x^4}{4} - \frac{5}{3}x^3 + 3x^2\right]_0^2 = \frac{64\pi}{3}$$



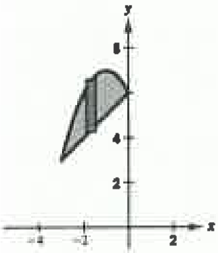
$2x - x^2,\ y = x + 6$ intersect at $(-3, 3)$ and $(0, 6)$.

$= 6 - 2x - x^2,\ r(x) = x + 6$

$$\pi\int_{-3}^{0} [(6 - 2x - x^2)^2 - (x + 6)^2]\,dx$$

$$\pi\int_{-3}^{0} (x^4 + 4x^3 - 9x^2 - 36x)\,dx$$

$$\pi\left[\frac{1}{5}x^5 + x^4 - 3x^3 - 18x^2\right]_{-3}^{0} = \frac{243\pi}{5}$$

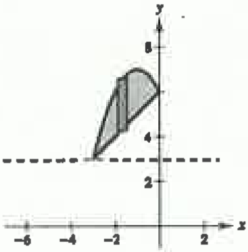


(b) $R(x) = (6 - 2x - x^2) - 3,\ r(x) = (x + 6) - 3$

$$V = \pi\int_{-3}^{0} [(3 - 2x - x^2)^2 - (x + 3)^2]\,dx$$

$$= \pi\int_{-3}^{0} (x^4 + 4x^3 - 3x^2 - 18x)\,dx$$

$$= \pi\left[\frac{1}{5}x^5 + x^4 - x^3 - 9x^2\right]_{-3}^{0} = \frac{108\pi}{5}$$

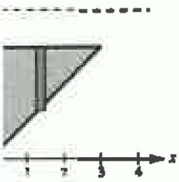


$4 - x,\ r(x) = 1$

$$\int_{0}^{3} [(4 - x)^2 - (1)^2]\,dx$$

$$\int_{0}^{3} (x^2 - 8x + 15)\,dx$$

$$\frac{x^3}{3} - 4x^2 + 15x\Big]_0^3 = 18\pi$$



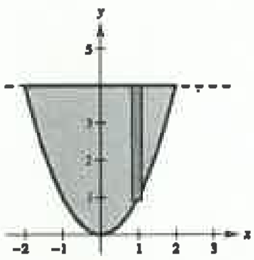
16. $R(x) = 4 - x^2,\ r(x) = 0$

$$V = 2\pi\int_{0}^{2} (4 - x^2)^2\,dx$$

$$= 2\pi\int_{0}^{2} (x^4 - 8x^2 + 16)\,dx$$

$$= 2\pi\left[\frac{x^5}{5} - \frac{8x^3}{3} + 16x\right]_0^2$$

$$= \frac{512\pi}{15}$$



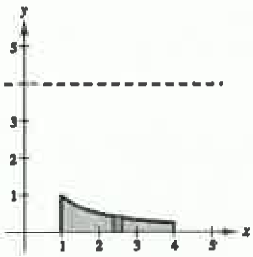
17. $R(x) = 4,\ r(x) = 4 - \dfrac{1}{x}$

$$V = \pi\int_{1}^{4} \left[(4)^2 - \left(4 - \frac{1}{x}\right)^2\right]dx$$

$$= \pi\int_{1}^{4} \left(\frac{8}{x} - \frac{1}{x^2}\right)dx$$

$$= \pi\left[8\ln|x| + \frac{1}{x}\right]_1^4$$

$$= \pi\left(8\ln 4 - \frac{3}{4}\right) \approx 32.49$$



$4,\ r(x) = 4 - \sec x$

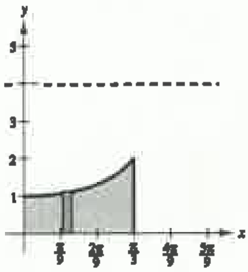
$$\int_{0}^{\pi/3} [(4)^2 - (4 - \sec x)^2]\,dx$$

$$\int_{0}^{\pi/3} (8\sec x - \sec^2 x)\,dx$$

$$8\ln|\sec x + \tan x| - \tan x\Big]_0^{\pi/3}$$

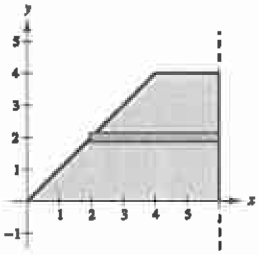
$$[8\ln|2 + \sqrt{3}| - \sqrt{3}) - (8\ln|1 + 0| - 0)]$$

$$8\ln(2 + \sqrt{3}) - \sqrt{3}] \approx 27.66$$



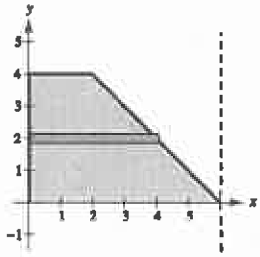
19. $R(y) = 6 - y,\ r(y) = 0$

$$V = \pi\int_0^4 (6-y)^2\,dy$$
$$= \pi\int_0^4 (y^2 - 12y + 36)\,dy$$
$$= \pi\left[\frac{y^3}{3} - 6y^2 + 36y\right]_0^4$$
$$= \frac{208\pi}{3}$$



20. $R(y) = 6,\ r(y) = 6 - (6 - y) = y$

$$V = \pi\int_0^4 [(6)^2 - (y)^2]\,dy$$
$$= \pi\left[36y - \frac{y^3}{3}\right]_0^4 = \frac{368\pi}{3}$$

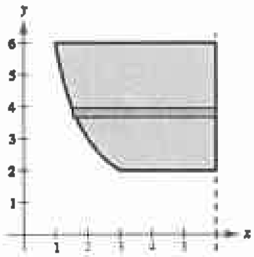


21. $R(y) = 6 - y^2,\ r(y)$

$$V = \pi\int_{-2}^2 [(6-y^2)^2$$
$$= 2\pi\int_0^2 (y^4 - 12y$$
$$= 2\pi\left[\frac{y^5}{5} - 4y^3 +\right.$$
$$= \frac{384\pi}{5}$$

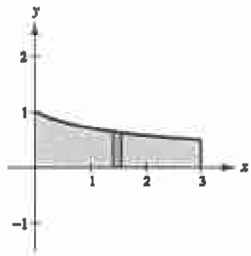
22. $R(y) = 6 - \frac{6}{y},\ r(y) = 0$

$$V = \pi\int_2^6 \left(6 - \frac{6}{y}\right)^2 dy$$
$$= 36\pi\int_2^6 \left(1 - \frac{2}{y} + \frac{1}{y^2}\right) dy$$
$$= 36\pi\left[y - 2\ln|y| - \frac{1}{y}\right]_2^6$$
$$= 36\pi\left[\left(\frac{35}{6} - 2\ln 6\right) - \left(\frac{3}{2} - 2\ln 2\right)\right]$$
$$= 36\pi\left(\frac{13}{3} + 2\ln\frac{1}{3}\right) = 12\pi(13 - 6\ln 3) \approx 241.59$$



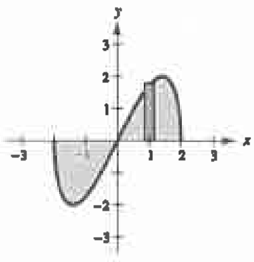
23. $R(x) = \frac{1}{\sqrt{x+1}},\ r(x) = 0$

$$V = \pi\int_0^3 \left(\frac{1}{\sqrt{x+1}}\right)^2 dx$$
$$= \pi\int_0^3 \frac{1}{x+1}\,dx$$
$$= \Big[\pi\ln|x+1|\Big]_0^3 = \pi\ln 4$$



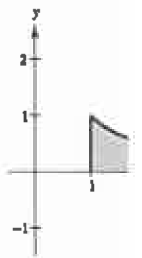
24. $R(x) = x\sqrt{4-x^2},\quad r(x) = 0$

$$V = 2\pi\int_0^2 \left[x\sqrt{4-x^2}\right]^2 dx$$
$$= 2\pi\int_0^2 (4x^2 - x^4)\,dx$$
$$= 2\pi\left[\frac{4x^3}{3} - \frac{x^5}{5}\right]_0^2$$
$$= \frac{128\pi}{15}$$



25. $R(x) = \frac{1}{x},\ r(x) = 0$

$$V = \pi\int_1^4 \left(\frac{1}{x}\right)^2 dx$$
$$= \pi\left[-\frac{1}{x}\right]_1^4$$
$$= \frac{3\pi}{4}$$



$\frac{3}{x+1}$, $r(x) = 0$

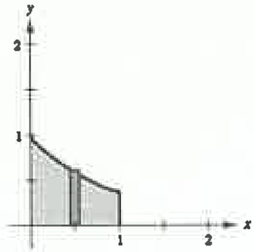
$\left(\frac{3}{x+1}\right)^2 dx$

$\int_0^8 (x+1)^{-2}\,dx$

$-\frac{1}{x+1}\Big]_0^8 = 8\pi$

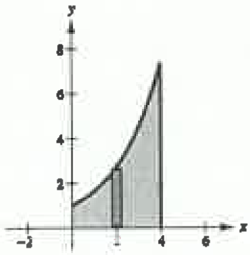
27. $R(x) = e^{-x}$, $r(x) = 0$

$$V = \pi\int_0^1 (e^{-x})^2\,dx$$
$$= \pi\int_0^1 e^{-2x}\,dx$$
$$= \left[-\frac{\pi}{2}e^{-2x}\right]_0^1$$
$$= \frac{\pi}{2}(1 - e^{-2}) \approx 1.358$$



28. $R(x) = e^{x/2}$, $r(x) = 0$

$$V = \pi\int_0^4 (e^{x/2})^2\,dx$$
$$= \pi\int_0^4 e^x\,dx$$
$$= \left[\pi e^x\right]_0^4$$
$$= \pi(e^4 - 1) \approx 168.38$$



$3x \implies x = \frac{1}{3}(6 - y)$

$\left[\frac{1}{3}(6 - y)\right]^2 dy$

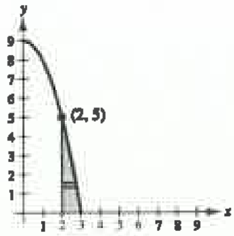
$[36 - 12y + y^2]\,dy$

$36y - 6y^2 + \frac{y^3}{3}\Big]_0^6$

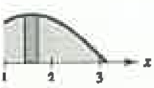
$216 - 216 + \frac{216}{3}\Big]$

30. $y = 9 - x^2$, $y = 0$, $x = 2$, $x = 3$

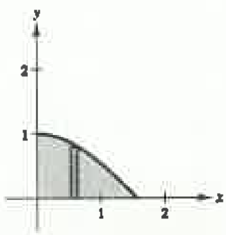
$$x = \sqrt{9 - y}$$
$$V = \int_2^5 \left[\left(\sqrt{9 - y}\right)^2 - 2^2\right] dy$$
$$= \int_2^5 (5 - y)\,dy$$
$$= \left[5y - \frac{y^2}{2}\right]_2^5$$
$$= \left(25 - \frac{25}{2}\right) - (10 - 2) = \frac{9}{2}$$



$\int_0^{\pi} [\sin x]^2\,dx \approx 4.9348$



32. $V = \pi\int_0^{\pi/2} [\cos x]^2\,dx \approx 2.4674$



33. $V = \pi \int_0^2 [e^{-x^2}]^2\, dx \approx 1.9686$

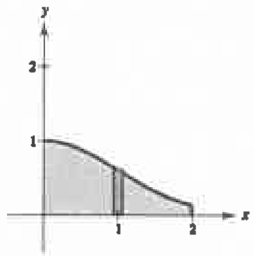
34. $V = \pi \int_1^3 [\ln x]^2\, dx \approx 3.2332$

35. $V = \pi \int_{-1}^2 [e^{x/2} + e^{-x/2}]^2\, dx \approx 49.0218$

36. $V = \pi \int_0^5 [2 \arctan (0.2x)]^2\, dx \approx 15.4115$

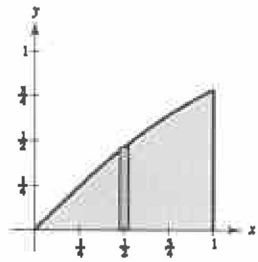
37. $A \approx 3$

Matches (a)



38. $A \approx \frac{3}{4}$

Matches (b)

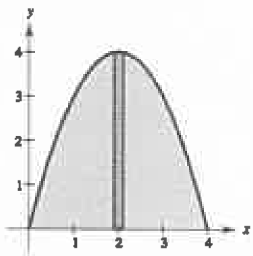


39. (a) $R(x) = 4x - x^2,\ r(x) = 0$

$$V = \pi \int_0^4 (4x - x^2)^2\, dx$$

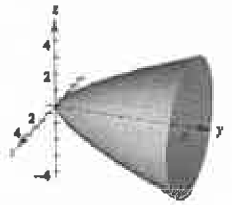
$$= \pi \int_0^4 (16x^2 - 8x^3 + x^4)\, dx$$

$$= \pi \left[\frac{16}{3}x^3 - 2x^4 + \frac{x^5}{5} \right]_0^4 = \frac{512\pi}{15}$$

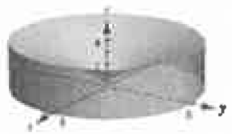


(b) Completing the square we have $4x - x^2 = 4 - (x^2 - 4x + 4) = 4 - (x - 2)^2$. Thus, $y = 4 - x^2$ has the sa as in part (a) since the solid has been translated only horizontally.

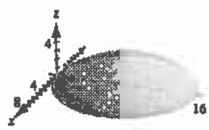
40. (a)



(b)



(c)



$a < c < b$.

41. $R(x) = \frac{1}{2}x,\ r(x) = 0$

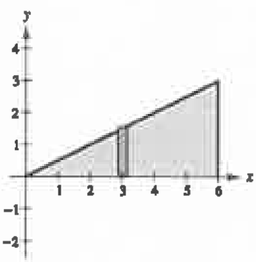
$$V = \pi \int_0^6 \frac{1}{4}x^2\, dx$$

$$= \left[\frac{\pi}{12}x^3 \right]_0^6 = 18\pi$$

Note: $V = \frac{1}{3}\pi r^2 h$

$$= \frac{1}{3}\pi(3^2)6$$

$$= 18\pi$$



42. $R(x) = \frac{r}{h}x,\ r(x) = 0$

$$V = \pi \int_0^h \frac{r^2}{h^2}x^2\, dx$$

$$= \left[\frac{r^2\pi}{3h^2}x^3 \right]_0^h$$

$$= \frac{r^2\pi}{3h^2}h^3 = \frac{1}{3}\pi r^2 h$$

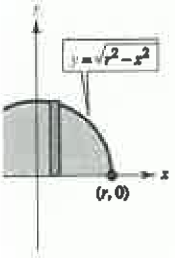
$\sqrt{r^2 - x^2},\ r(x) = 0$

$(r^2 - x^2)\,dx$

$(r^2 - x^2)\,dx$

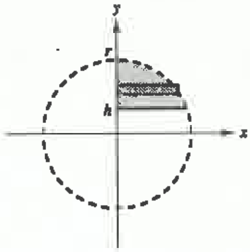
$r^2x - \frac{1}{3}x^3\Big]_0^r$

$r^3 - \frac{1}{3}r^3\Big) = \frac{4}{3}\pi r^3$



44. $x = \sqrt{r^2 - y^2},\ R(y) = \sqrt{r^2 - y^2},\ r(y) = 0$

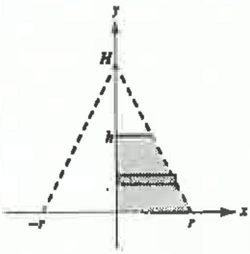
$$V = \pi\int_h^r\left(\sqrt{r^2 - y^2}\right)^2 dy$$
$$= \pi\int_h^r (r^2 - y^2)\,dy$$
$$= \pi\left[r^2y - \frac{y^3}{3}\right]_h^r$$
$$= \pi\left[\left(r^3 - \frac{r^3}{3}\right) - \left(r^2h - \frac{h^3}{3}\right)\right]$$
$$= \pi\left(\frac{2r^3}{3} - r^2h + \frac{h^3}{3}\right)$$
$$= \frac{\pi}{3}(2r^3 - 3r^2h + h^3)$$



$\frac{r}{H}y = r\left(1 - \frac{y}{H}\right),\ R(y) = r\left(1 - \frac{y}{H}\right),\ r(y) = 0$

$\left[r\left(1 - \frac{y}{H}\right)\right]^2 dy = \pi r^2\int_0^h\left(1 - \frac{2}{H}y + \frac{1}{H^2}y^2\right)dy$

$$= \pi r^2\left[y - \frac{1}{H}y^2 + \frac{1}{3H^2}y^3\right]_0^h$$
$$= \pi r^2\left(h - \frac{h^2}{H} + \frac{h^3}{3H^2}\right)$$
$$= \pi r^2h\left(1 - \frac{h}{H} + \frac{h^2}{3H^2}\right)$$



$\pi\int_0^4(\sqrt{x})^2\,dx = \pi\int_0^4 x\,dx = \left[\frac{\pi x^2}{2}\right]_0^4 = 8\pi$

$0 < c < 4$ and set

$\pi\int_0^c x\,dx = \left[\frac{\pi x^2}{2}\right]_0^c = \frac{\pi c^2}{2} = 4\pi.$

$c^2 = 8$

$c = \sqrt{8} = 2\sqrt{2}$

s, when $x = 2\sqrt{2}$, the solid is divided into two s of equal volume.

(b) Set $\pi\int_0^c x\,dx = \frac{8\pi}{3}$ (one third of the volume). Then

$$\frac{\pi c^2}{2} = \frac{8\pi}{3},\ c^2 = \frac{16}{3},\ c = \frac{4}{\sqrt{3}} = \frac{4\sqrt{3}}{3}.$$

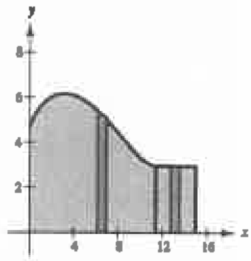
To find the other value, set $\pi\int_0^d x\,dx = \frac{16\pi}{3}$ (two thirds of the volume). Then

$$\frac{\pi d^2}{2} = \frac{16\pi}{3},\ d^2 = \frac{32}{3},\ d = \frac{\sqrt{32}}{\sqrt{3}} = \frac{4\sqrt{6}}{3}.$$

The x-values that divide the solid into three parts of equal volume are $x = (4\sqrt{3})/3$ and $x = (4\sqrt{6})/3$.

47. $V = \pi\int_0^2\left(\frac{1}{8}x^2\sqrt{2-x}\right)^2 dx = \frac{\pi}{64}\int_0^2 x^4(2-x)\,dx = \frac{\pi}{64}\left[\frac{2x^5}{5} - \frac{x^6}{6}\right]_0^2 = \frac{\pi}{30}$

48. $y = \begin{cases} \sqrt{0.1x^3 - 2.2x^2 + 10.9x + 22.2}, & 0 \le x \le 11.5 \\ 2.95, & 11.5 < x \le 15 \end{cases}$



$$V = \pi\int_0^{11.5}\left(\sqrt{0.1x^3 - 2.2x^2 + 10.9x + 22.2}\right)^2 dx + \pi\int_{11.5}^{15} 2.95^2\,dx$$

$$= \pi\left[\frac{0.1x^4}{4} - \frac{2.2x^3}{3} + \frac{10.9x^2}{2} + 22.2x\right]_0^{11.5} + \pi\left[2.95^2 x\right]_{11.5}^{15}$$

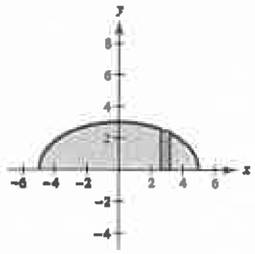
$\approx$ 1031.9016 cubic centimeters

49. (a) $R(x) = \frac{3}{5}\sqrt{25 - x^2},\ r(x) = 0$

$$V = \frac{9\pi}{25}\int_{-5}^{5}(25 - x^2)\,dx$$

$$= \frac{18\pi}{25}\int_0^5 (25 - x^2)\,dx$$

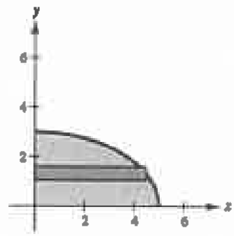
$$= \frac{18\pi}{25}\left[25x - \frac{x^3}{3}\right]_0^5 = 60\pi$$



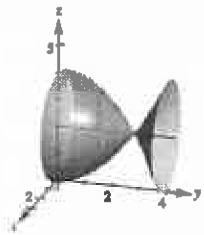
(b) $R(y) = \frac{5}{3}\sqrt{9 - y^2},\ r(y) = 0,\ x \ge 0$

$$V = \frac{25\pi}{9}\int_0^3 (9 - y^2)\,dy$$

$$= \frac{25\pi}{9}\left[9y - \frac{y^3}{3}\right]_0^3 = 50\pi$$



50. (a) First find where $y = b$ intersects the parabola:



$$b = 4 - \frac{x^2}{4}$$

$$x^2 = 16 - 4b = 4(4 - b)$$

$$x = 2\sqrt{4 - b}$$

$$V = \int_0^{2\sqrt{4-b}} \pi\left[4 - \frac{x^2}{4} - b\right]^2 dx + \int_{2\sqrt{4-b}}^{4} \pi\left[b - 4 + \frac{x^2}{4}\right]^2 dx$$

$$= \int_0^4 \pi\left[4 - \frac{x^2}{4} - b\right]^2 dx$$

$$= \pi\int_0^4 \left[\frac{x^4}{16} - 2x^2 + \frac{bx^2}{2} + b^2 - 8b + 16\right] dx$$

$$= \pi\left[\frac{x^5}{80} - \frac{2x^3}{3} + \frac{bx^3}{6} + b^2x - 8bx + 16x\right]_0^4$$

$$= \pi\left[\frac{64}{5} - \frac{128}{3} + \frac{32}{3}b + 4b^2 - 32b + 64\right]$$

$$= \pi\left[4b^2 - \frac{64}{3}b + \frac{512}{15}\right]$$

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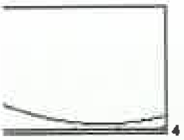
h of $V(b) = \pi\left[4b^2 - \frac{64}{3}b + \frac{512}{15}\right]$

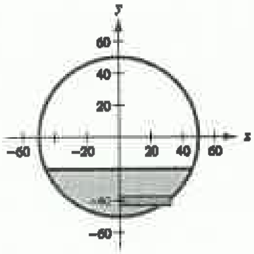
(c) $V'(b) = \pi\left[8b - \frac{64}{3}\right] = 0 \Rightarrow b = \frac{64/3}{8} = \frac{8}{3} = 2\frac{2}{3}$

$V''(b) = 8\pi > 0 \Rightarrow b = \frac{8}{3}$ is a relative minimum.



imum Volume is 17.87 for $b = 2.67$

lume: $V = \frac{4\pi(50)^3}{3} = \frac{500,000}{3}$ ft^3



of water in the tank:

$$\int_{-50}^{y_0} (\sqrt{2500 - y^2})^2\,dy = \pi\int_{-50}^{y_0} (2500 - y^2)\,dy$$

$$= \pi\left[2500y - \frac{y^3}{3}\right]_{-50}^{y_0}$$

$$= \pi\left(2500y_0 - \frac{y_0^3}{3} + \frac{250,000}{3}\right)$$

ne tank is one-fourth of its capacity:

$$\frac{1}{4}\left(\frac{500,000\pi}{3}\right) = \pi\left(2500y_0 - \frac{y_0^3}{3} + \frac{250,00}{3}\right)$$

$$125,000 = 7500y_0 - y_0^3 + 250,000$$

$- 7500y_0 - 125,000 = 0$

$$y_0 \approx -17.36$$

$-17.36 - (-50) = 32.64$ feet

ne tank is three-fourths of its capacity the depth is $100 - 32.64 = 67.36$ feet.

$= \int_0^{10} \pi[f(x)]^2\,dx$

npson's Rule: $b - a = 10 - 0 = 10, \quad n = 10$

$= \frac{\pi}{3}[(2.1)^2 + 4(1.9)^2 + 2(2.1)^2 + 4(2.35)^2 + 2(2.6)^2 + 4(2.85)^2 + 2(2.9)^2 + 4(2.7)^2 + 2(2.45)^2 + 4(2.2)^2 + (2.3)^2]$

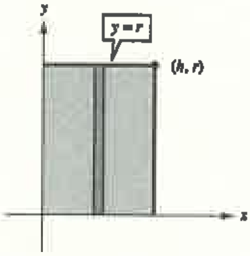
$= \frac{\pi}{3}[178.405] \approx 186.83$ cm^3

$) = 0.00249x^4 - 0.0529x^3 + 0.3314x^2 - 0.4999x + 2.112$

$= \int_0^{10} \pi f(x)^2\,dx \approx 186.35$ cm^3

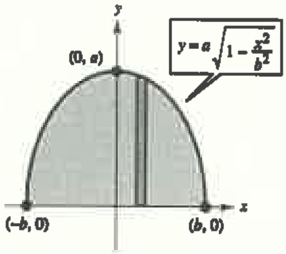
53. (a) $\pi\int_0^h r^2\,dx$ (ii)

is the volume of a right circular cylinder with radius r and height h.



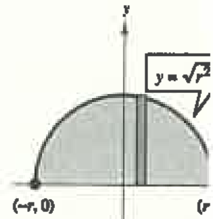
(b) $\pi\int_{-b}^{b}\left(a\sqrt{1-\frac{x^2}{b^2}}\right)^2 dx$ (iv)

is the volume of an ellipsoid with axes $2a$ and $2b$.



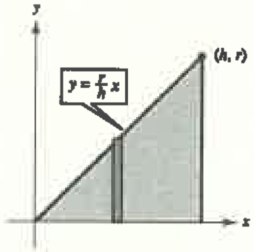
(c) $\pi\int_{-r}^{r}\left(\sqrt{r^2-x^2}\right)^2 dx$

is the volume of a sp
radius r.



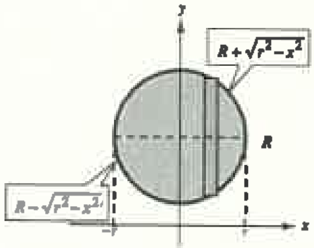
(d) $\pi\int_0^h\left(\frac{rx}{h}\right)^2 dx$ (i)

is the volume of a right circular cone with the radius of the base as r and height h.



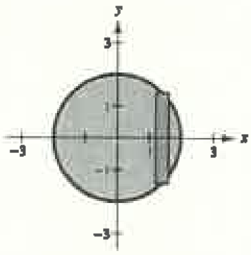
(e) $\pi\int_{-r}^{r}\left[\left(R+\sqrt{r^2-x^2}\right)^2-\left(R-\sqrt{r^2-x^2}\right)\right.$

is the volume of a torus with the radius of its
section as r and the distance from the axis of
the center of its cross section as R.



54. $V = \frac{1}{2}(8)(1)(2) = 8\text{ m}^3$

55.

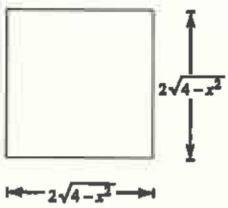


Base of Cross Section $= 2\sqrt{4-x^2}$

(a) $A(x) = b^2 = \left(2\sqrt{4-x^2}\right)^2$

$$V = \int_{-2}^{2} 4(4-x^2)\,dx$$

$$= 4\left[4x-\frac{x^3}{3}\right]_{-2}^{2} = \frac{128}{3}$$

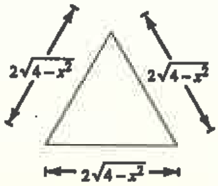


(b) $A(x) = \frac{1}{2}bh = \frac{1}{2}\left(2\sqrt{4-x^2}\right)\left(\sqrt{3}\sqrt{4-x^2}\right)$

$$= \sqrt{3}\,(4-x^2)$$

$$V = \sqrt{3}\int_{-2}^{2}(4-x^2)\,dx$$

$$= \sqrt{3}\left[4x-\frac{x^3}{3}\right]_{-2}^{2} = \frac{32\sqrt{3}}{3}$$



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$= \frac{1}{2}\pi r^2$

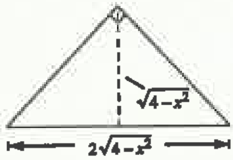
$= \frac{\pi}{2}(\sqrt{4-x^2})^2 = \frac{\pi}{2}(4-x^2)$

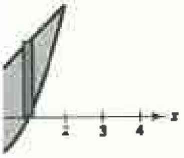
$\frac{\pi}{2}\int_{-2}^{2}(4-x^2)\,dx = \frac{\pi}{2}\left[4x - \frac{x^3}{3}\right]_{-2}^{2} = \frac{16\pi}{3}$

(d) $A(x) = \frac{1}{2}bh$

$= \frac{1}{2}(2\sqrt{4-x^2})(\sqrt{4-x^2}) = 4 - x^2$

$V = \int_{-2}^{2}(4-x^2)\,dx = \left[4x - \frac{x^3}{3}\right]_{-2}^{2} = \frac{32}{3}$





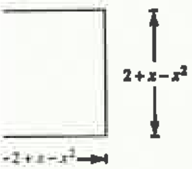
Cross Section $= (x+1) - (x^2-1) = 2 + x - x^2$

$) = b^2 = (2 + x - x^2)^2$

$= 4 + 4x - 3x^2 - 2x^3 + x^4$

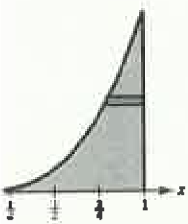
$\int_{-1}^{2}(4 + 4x - 3x^2 - 2x^3 + x^4)\,dx$

$= \left[4x + 2x^2 - x^3 - \frac{1}{2}x^4 + \frac{1}{5}x^5\right]_{-1}^{2} = \frac{81}{10}$



(b) $A(x) = bh = (2 + x - x^2)1$

$V = \int_{-1}^{2}(2 + x - x^2)\,dx = \left[2x + \frac{x^2}{2} - \frac{x^3}{3}\right]_{-1}^{2} = \frac{9}{2}$



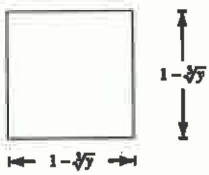
Cross Section $= 1 - \sqrt[3]{y}$

(a) $A(y) = b^2 = (1 - \sqrt[3]{y})^2$

$V = \int_0^1 (1 - \sqrt[3]{y})^2\,dy$

$= \int_0^1 (1 - 2y^{1/3} + y^{2/3})\,dy$

$= \left[y - \frac{3}{2}y^{4/3} + \frac{3}{5}y^{5/3}\right]_0^1 = \frac{1}{10}$



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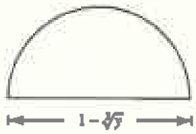
(b) $A(y) = \frac{1}{2}\pi r^2 = \frac{1}{2}\pi\left(\frac{1-\sqrt[3]{y}}{2}\right)^2 = \frac{1}{8}\pi(1-\sqrt[3]{y})^2$

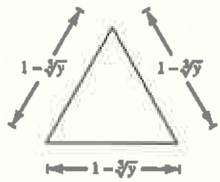
$$V = \frac{1}{8}\pi\int_0^1 (1-\sqrt[3]{y})^2\,dy = \frac{\pi}{8}\left(\frac{1}{10}\right) = \frac{\pi}{80}$$



(c) $A(y) = \frac{1}{2}bh = \frac{1}{2}(1-\sqrt[3]{y})\left(\frac{\sqrt{3}}{2}\right)(1-\sqrt[3]{y})$

$$= \frac{\sqrt{3}}{4}(1-\sqrt[3]{y})^2$$

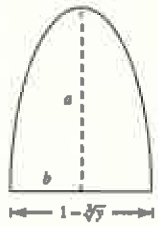
$$V = \frac{\sqrt{3}}{4}\int_0^1 (1-\sqrt[3]{y})^2\,dy = \frac{\sqrt{3}}{4}\left(\frac{1}{10}\right) = \frac{\sqrt{3}}{40}$$



(d) $A(y) = \frac{1}{2}\pi ab = \frac{\pi}{2}(2)(1-\sqrt[3]{y})\frac{1-\sqrt[3]{y}}{2}$

$$= \frac{\pi}{2}(1-\sqrt[3]{y})^2$$

$$V = \frac{\pi}{2}\int_0^1 (1-\sqrt[3]{y})^2\,dy = \frac{\pi}{2}\left(\frac{1}{10}\right) = \frac{\pi}{20}$$



58. The cross sections are squares. By symmetry, we can set up an integral for an eighth of the volume and multiply by 8.

$$A(y) = b^2 = \left(\sqrt{r^2-y^2}\right)^2$$

$$V = 8\int_0^r (r^2-y^2)\,dy$$

$$= 8\left[r^2y - \frac{1}{3}y^3\right]_0^r$$

$$= \frac{16}{3}r^3$$

59. Assume that the oil just hits the top of the cyl means that the volume of oil is one-half of th the cylinder. Then we have

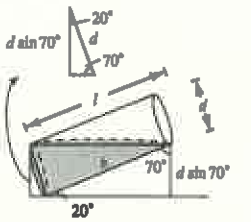
$$\sin 20^\circ = \frac{d\sin 70^\circ}{h}$$

$$h = \frac{d\sin 70^\circ}{\sin 20^\circ}$$

$$\text{Volume} = \frac{1}{2}(\pi r^2 h)$$

$$= \frac{\pi}{2}\left(\frac{d}{2}\right)^2\left(\frac{d\sin 70^\circ}{\sin 20^\circ}\right)$$

$$= \frac{\pi d^3 \sin 70^\circ}{8\sin 20^\circ} = \pi d^3 \cot 20^\circ.$$

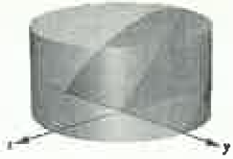


60. Let $A_1(x)$ and $A_2(x)$ equal the areas of the cross sections of the two solids for $a \le x \le b$. Since $A_1(x) = A_2(x)$, we

$$V_1 = \int_a^b A_1(x)\,dx = \int_a^b A_2(x)\,dx = V_2$$

Thus, the volumes are the same.

ce the cross sections are isosceles right triangles:



$$A(x) = \frac{1}{2}bh = \frac{1}{2}\left(\sqrt{r^2 - y^2}\right)\left(\sqrt{r^2 - y^2}\right) = \frac{1}{2}(r^2 - y^2)$$

$$V = \frac{1}{2}\int_{-r}^{r}(r^2 - y^2)\,dy = \int_0^r (r^2 - y^2)\,dy = \left[r^2y - \frac{y^3}{3}\right]_0^r = \frac{2}{3}r^3$$

$$) = \frac{1}{2}bh = \frac{1}{2}\sqrt{r^2 - y^2}\left(\sqrt{r^2 - y^2}\tan\theta\right) = \frac{\tan\theta}{2}(r^2 - y^2)$$

$$= \frac{\tan\theta}{2}\int_{-r}^{r}(r^2 - y^2)\,dy = \tan\theta\int_0^r (r^2 - y^2)\,dy = \tan\theta\left[r^2y - \frac{y^3}{3}\right]_0^r = \frac{2}{3}r^3\tan\theta$$

$\theta \to 90°$, $V \to \infty$.

$$\int_{-\sqrt{R^2-r^2}}^{\sqrt{R^2-r^2}} \left[\left(\sqrt{R^2 - x^2}\right)^2 - r^2\right] dx$$

$$\pi\int_0^{\sqrt{R^2-r^2}} (R^2 - r^2 - x^2)\,dx$$

$$\pi\left[(R^2 - r^2)x - \frac{x^3}{3}\right]_0^{\sqrt{R^2-r^2}}$$

$$\pi\left[(R^2 - r^2)^{3/2} - \frac{(R^2 - r^2)^{3/2}}{3}\right]$$

$$\pi(R^2 - r^2)^{3/2}$$

63. $$\frac{4}{3}\pi(25 - r^2)^{3/2} = \frac{1}{2}\left(\frac{4}{3}\right)\pi(125)$$

$$(25 - r^2)^{3/2} = \frac{125}{2}$$

$$25 - r^2 = \left(\frac{125}{2}\right)^{2/3}$$

$$25 - \frac{25}{(2^{2/3})} = r^2$$

$$25(1 - 2^{-2/3}) = r^2$$

$$r = 5\sqrt{1 - 2^{-2/3}} \approx 3.0415$$

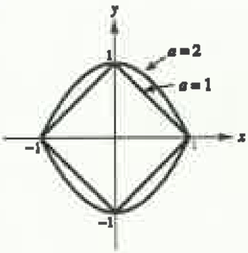
hen $a = 1$: $|x| + |y| = 1$ represents a square.

hen $a = 2$: $|x|^2 + |y|^2 = 1$ represents a circle.

$$= (1 - |x|^a)^{1/a}$$

$$= 2\int_{-1}^{1}(1 - |x|^a)^{1/a}\,dx = 4\int_0^1 (1 - x^a)^{1/a}\,dx$$

approximate the volume of the solid, form n slices, each of whose area is proximated by the integral above. Then sum the volumes of these n slices.



6.3 Volume: The Shell Method

x

x

$$\pi\int_0^2 x(x)\,dx = \left[\frac{2\pi x^3}{3}\right]_0^2 = \frac{16\pi}{3}$$

2. $p(x) = x$

$h(x) = 1 - x$

$$V = 2\pi\int_0^1 x(1 - x)\,dx$$

$$= 2\pi\int_0^1 (x - x^2)\,dx = 2\pi\left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1 = \frac{\pi}{3}$$