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SECTION PROJECT

St. Louis Arch The Gateway Arch in St. Louis, Missouri was constructed using the hyperbolic cosine function. The equation used to construct the arch was

$$y = 693.8597 - 68.7672 \cosh 0.0100333x,$$
$$-299.2239 \leq x \leq 299.2239$$

where x and y are measured in feet. Cross sections of the arch are equilateral triangles and (x, y) traces the path of the centers of mass of the cross-sectional triangles. For each value of x, the area of the cross-sectional triangle is

$$A = 125.1406 \cosh 0.0100333x.$$

(Source: Owner's Manual for the Gateway Arch, *Saint Louis, MO, by William Thayer)*

(a) How high above the ground is the center of the highest triangle? (At ground level, $y = 0$.)

(b) What is the height of the arch? (*Hint:* For an equilateral triangle, $A = \sqrt{3}c^2$, where c is one-half the base of the triangle, and the center of mass of the triangle is located at two-thirds the height of the triangle.)

(c) How wide is the arch at ground level?

REVIEW EXERCISES FOR CHAPTER 5

In Exercises 1 and 2, sketch the graph of the function by hand. Identify any asymptotes of the graph.

1. $f(x) = \ln x + 3$

2. $f(x) = \ln(x - 3)$

In Exercises 3 and 4, use the properties of logarithms to write the expression as a sum, difference, and/or multiple of logarithms.

3. $\ln \sqrt[5]{\dfrac{4x^2 - 1}{4x^2 + 1}}$

4. $\ln[(x^2 + 1)(x - 1)]$

In Exercises 5 and 6, write the expression as the logarithm of a single quantity.

5. $\ln 3 + \frac{1}{3} \ln(4 - x^2) - \ln x$

6. $3[\ln x - 2 \ln(x^2 + 1)] + 2 \ln 5$

True or False? **In Exercises 7 and 8, determine whether the statement is true or false.**

7. The domain of the function $f(x) = \ln x$ is the set of all real numbers.

8. $\ln(x + y) = \ln x + \ln y$

In Exercises 9 and 10, solve the equation for x.

9. $\ln \sqrt{x + 1} = 2$

10. $\ln x + \ln(x - 3) = 0$

In Exercises 11–18, find the derivative of the function.

11. $g(x) = \ln \sqrt{x}$

12. $h(x) = \ln \dfrac{x(x - 1)}{x - 2}$

13. $f(x) = x\sqrt{\ln x}$

14. $f(x) = \ln[x(x^2 - 2)^{2/3}]$

15. $y = \dfrac{1}{b^2}\left[\ln(a + bx) + \dfrac{a}{a + bx}\right]$

16. $y = \dfrac{1}{b^2}[a + bx - a \ln(a + bx)]$

17. $y = -\dfrac{1}{a} \ln \dfrac{a + bx}{x}$

18. $y = -\dfrac{1}{ax} + \dfrac{b}{a^2} \ln \dfrac{a + bx}{x}$

In Exercises 19–26, evaluate the integral.

19. $\displaystyle\int \frac{1}{7x - 2}\, dx$

20. $\displaystyle\int \frac{x}{x^2 - 1}\, dx$

21. $\displaystyle\int \frac{\sin x}{1 + \cos x}\, dx$

22. $\displaystyle\int \frac{\ln \sqrt{x}}{x}\, dx$

23. $\displaystyle\int_1^4 \frac{x + 1}{x}\, dx$

24. $\displaystyle\int_1^e \frac{\ln x}{x}\, dx$

25. $\displaystyle\int_0^{\pi/3} \sec \theta \, d\theta$

26. $\displaystyle\int_0^{\pi/4} \tan\left(\frac{\pi}{4} - x\right) dx$

In Exercises 27–34, (a) find the inverse of the function, (b) use a graphing utility to graph f and f^{-1} in the same viewing rectangle, and (c) verify that $f^{-1}(f(x)) = f(f^{-1}(x)) = x$.

27. $f(x) = \frac{1}{2}x - 3$
28. $f(x) = 5x - 7$
29. $f(x) = \sqrt{x + 1}$
30. $f(x) = x^3 + 2$
31. $f(x) = \sqrt[3]{x + 1}$
32. $f(x) = x^2 - 5, \quad x \geq 0$
33. $f(x) = \ln \sqrt{x}$
34. $f(x) = e^{1-x}$

In Exercises 35–38, sketch the graph of the function by hand.

35. $y = e^{-x/2}$
36. $g(x) = 6(2^{-x^2})$
37. $h(x) = -3 \arcsin 2x$
38. $f(x) = 2 \arctan(x + 3)$

In Exercises 39 and 40, evaluate the expression without using a calculator. (*Hint:* Make a sketch of a right triangle.)

39. (a) $\sin\left(\arcsin \frac{1}{2}\right)$
(b) $\cos\left(\arcsin \frac{1}{2}\right)$

40. (a) $\tan(\text{arccot } 2)$
(b) $\cos\left(\text{arcsec } \sqrt{5}\right)$

In Exercises 41–58, find the derivative of the function.

41. $f(x) = \ln(e^{-x^2})$
42. $g(x) = \ln \dfrac{e^x}{1 + e^x}$
43. $g(t) = t^2e^t$
44. $h(z) = e^{-z^2/2}$
45. $y = \sqrt{e^{2x} + e^{-2x}}$
46. $y = x^{2x+1}$
47. $f(x) = 3^{x-1}$
48. $f(x) = (4e)^x$
49. $g(x) = \dfrac{x^2}{e^x}$
50. $f(\theta) = \dfrac{1}{2}e^{\sin 2\theta}$
51. $y = \tan(\arcsin x)$
52. $y = \arctan(x^2 - 1)$
53. $y = x \text{ arcsec } x$
54. $y = \frac{1}{2} \arctan e^{2x}$
55. $y = x(\arcsin x)^2 - 2x + 2\sqrt{1 - x^2} \arcsin x$
56. $y = \sqrt{x^2 - 4} - 2 \text{ arcsec}(x/2), \quad 2 < x < 4$
57. $y = 2x - \cosh \sqrt{x}$
58. $y = x \tanh^{-1} 2x$

In Exercises 59 and 60, use implicit differentiation to find dy/dx.

59. $y \ln x + y^2 = 0$
60. $\cos x^2 = xe^y$

61. ***Think About It*** Find the derivative of each function, given that a is constant.
(a) $y = x^a$, (b) $y = a^x$, (c) $y = x^x$, (d) $y = a^a$

62. ***Compound Interest*** How large a deposit, at 7 percent interest compounded continuously, must be made to obtain a balance of \$10,000 in 15 years?

63. ***Compound Interest*** A deposit earns interest at a rate of r percent compounded continuously and doubles in value in 10 years. Find r.

64. ***Climb Rate*** The time t (in minutes) for a small plane to climb to an altitude of h feet is

$$t = 50 \log_{10} \frac{18{,}000}{18{,}000 - h}$$

where 18,000 feet is the plane's absolute ceiling.
(a) Determine the domain of the function appropriate for the context of the problem.
(b) Use a graphing utility to graph the time function and identify any asymptotes.
(c) As the plane approaches its absolute ceiling, what can be concluded about the time required to further increase its altitude?
(d) Find the time when the altitude is increasing at the greatest rate.

In Exercises 65–80, evaluate the integral.

65. $\displaystyle\int xe^{-3x^2}\, dx$
66. $\displaystyle\int \frac{e^{1/x}}{x^2}\, dx$
67. $\displaystyle\int \frac{e^{4x} - e^{2x} + 1}{e^x}\, dx$
68. $\displaystyle\int \frac{e^{2x} - e^{-2x}}{e^{2x} + e^{-2x}}\, dx$
69. $\displaystyle\int \frac{e^x}{e^x - 1}\, dx$
70. $\displaystyle\int x^2e^{x^3+1}\, dx$
71. $\displaystyle\int \frac{1}{e^{2x} + e^{-2x}}\, dx$
72. $\displaystyle\int \frac{1}{3 + 25x^2}\, dx$
73. $\displaystyle\int \frac{x}{\sqrt{1 - x^4}}\, dx$
74. $\displaystyle\int \frac{1}{16 + x^2}\, dx$
75. $\displaystyle\int \frac{x}{16 + x^2}\, dx$
76. $\displaystyle\int \frac{4 - x}{\sqrt{4 - x^2}}\, dx$
77. $\displaystyle\int \frac{\arctan(x/2)}{4 + x^2}\, dx$
78. $\displaystyle\int \frac{\arcsin x}{\sqrt{1 - x^2}}\, dx$
79. $\displaystyle\int \frac{x}{\sqrt{x^4 - 1}}\, dx$
80. $\displaystyle\int x^2 \text{ sech}^2\, x^3\, dx$

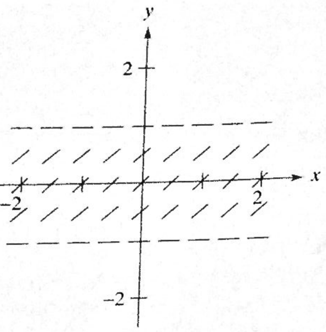
In Exercises 81 and 82, find the area of the region bounded by the graphs of the equations.

81. $y = xe^{-x^2}, \quad y = 0, \quad x = 0, \quad x = 4$
82. $y = \dfrac{1}{x^2 + 1}, \quad y = 0, \quad x = 0, \quad x = 1$

In Exercises 83–88, solve the differential equation.

83. $\dfrac{dy}{dx} = \dfrac{x^2 + 3}{x}$
84. $\dfrac{dy}{dx} = \dfrac{e^{-2x}}{1 + e^{-2x}}$
85. $y' - 2xy = 0$
86. $y' - e^y \sin x = 0$
87. $\dfrac{dy}{dx} = \dfrac{x^2 + y^2}{2xy}$
88. $\dfrac{dy}{dx} = \dfrac{3(x + y)}{x}$

ield shown in the figure to answer ea



) Sketch several solution curves to the differential equation on the direction field.

) When is the rate of change of the solution the greatest? When is it the least?

Find the general solution of the differential equation. Compare the result with the sketches in part (a).

ow that $y = e^x(a \cos 3x + b \sin 3x)$ satisfies the differential uation $y'' - 2y' + 10y = 0$.

d the orthogonal trajectories of the family $(x - C)^2 +$ $= C^2$ and use a graphing utility to sketch several members he family.

Pressure Under ideal conditions, air pressure decreases tinuously with height above sea level at a rate proportional he pressure at that height. If the barometer reads 30 inches ea level and 15 inches at 18,000 feet, find the barometric ssure at 35,000 feet.

bability Two numbers between 0 and 10 are chosen at om. The probability that their product is less than $n(0 <$ $100)$ is

$$\frac{1}{100}\left(n + \int_{n/10}^{10} \frac{n}{x}\,dx\right).$$

Vhat is the probability that the product is less than 25?

Vhat is the probability that the product is less than 50?

cal Motion A falling object encounters air resistance s proportional to its velocity. If the acceleration due to y is considered to be -9.8 meters per second per second, t change in velocity is

$kv - 9.8$.

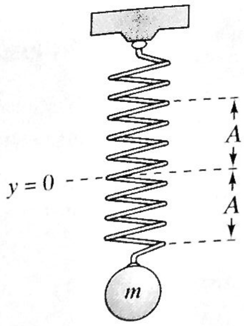
nd the velocity of the object as a function of time if the tial velocity is v_0.

e the result in part (a) to find the limit of the velocity as pproaches infinity.

egrate the velocity function found in part (a) to find the ition function s.

$$\int \frac{dy}{\sqrt{A^2 - y^2}} = \int \sqrt{\frac{k}{m}}\,dt$$

where A is the maximum displacement, t is the time, and k is a constant. Find y as a function of t, given that $y = 0$ when t



96. *Modeling Data* For the years 1850 through 1990, the ordered pairs (t, P) give the population of the United States (in millions), where t is the time in years, with $t = 0$ corresponding to 1900. *(Source: U.S. Bureau of Census)*

$(-50, 23.2)$, $(-40, 31.4)$, $(-30, 39.8)$, $(-20, 50.2)$,

$(-10, 62.9)$, $(0, 76.0)$, $(10, 92.0)$, $(20, 105.7)$,

$(30, 122.8)$, $(40, 131.7)$, $(50, 150.7)$, $(60, 178.5)$,

$(70, 203.3)$, $(80, 226.5)$, $(90, 248.7)$

(a) Use the regression capabilities of a graphing utility to find the following models for the data.

Linear: $y = at + b$

Quadratic: $y = at^2 + bt + c$

Exponential: $y = ab^t$

(b) Use a graphing utility to plot the data and graph each of the models.

(c) Evaluate each model and the rate of growth of each model for the year 1998. Which model seems to be the best?

97. *Fuel Economy* A certain automobile gets 28 miles per gallon of gasoline for speeds up to 50 miles per hour. Over 50 miles per hour, the number of miles per gallon drops at the rate of 12 percent for each 10 miles per hour.

(a) If s is the speed and y is the number of miles per gallon, find y as a function of s by solving the differential equation

$$\frac{dy}{ds} = -0.012y, \quad s > 50.$$

(b) Use the function in part (a) to complete the table.

Speed	50	55	60	65	70
Miles per gallon					