APCalc
Mr. C. Sdoutz

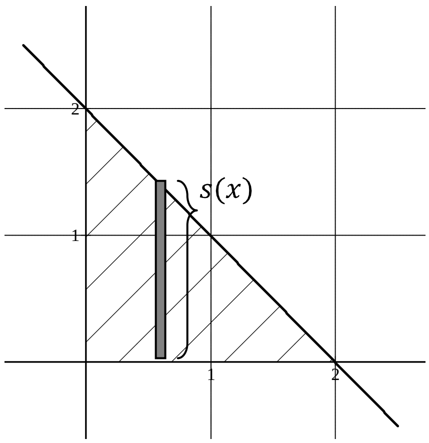
Date:________________

Unit 6: Applications of Integration
6.7 Volume of Known Cross-Sections

Solids that are not created through revolution but we know the base region and the constant cross-section shape. [applet1] [applet2] [applet3][applet4]

We can find the volumes of these solids by adding together the volumes of the slices that make up the solid (like we did using discs for solids of revolution).

ex. Find the volume of a shape whose base region is bounded by $y = -x + 2, y = 0$ & $x = 0$ and whose cross-sections perpendicular to the x-axis are squares.



Area of square cross-sections $= s(x)^2$

$s(x) = -x + 2$

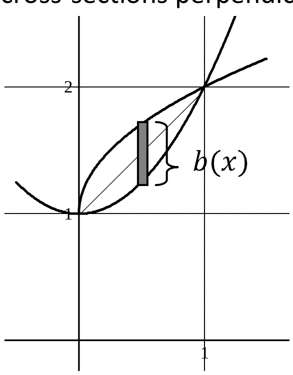
Volume $=$ sum of the cross section areas

$$V = \int_a^b s(x)^2\,dx$$

$$= \int_0^2 (-x+2)^2\,dx$$

$$\boxed{= \frac{8}{3}\,un^3}$$

ex. Find the volume of a shape whose base region is bounded by $y = \sqrt{x} + 1$ & $y = x^2 + 1$ and whose cross-sections perpendicular to the x-axis are rectangles of height 6 units.



Area of rectangular cross-sections $= b(x) \cdot h$

$b(x) = (\sqrt{x} + 1) - (x^2 + 1) = \sqrt{x} - x^2$

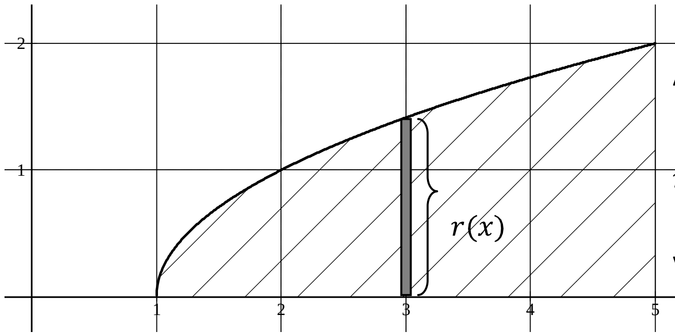
Volume $=$ sum of the cross section areas

$$V = h\int_a^b b(x)\,dx$$

$$= 6\int_0^1 \sqrt{x} - x^2\,dx$$

$$\boxed{= 2\,un^3}$$

ex. Find the volume of a shape whose base region is bounded by $y = \sqrt{x-1}, y = 0$ & $x = 5$ and whose cross-sections perpendicular to the x-axis are semi-circles.



Area of semi-circular cross-sections $= \frac{\pi r(x)^2}{2}$

$r(x) = \frac{\sqrt{x-1}}{2}$

Volume = sum of the cross section areas

$$V = \frac{\pi}{2}\int_a^b r(x)^2\, dx$$

$$= \frac{\pi}{2}\int_1^5 \left(\frac{\sqrt{x-1}}{2}\right)^2 dx$$

$$\boxed{= \pi\ un^3}$$

Practice Questions

Worksheet