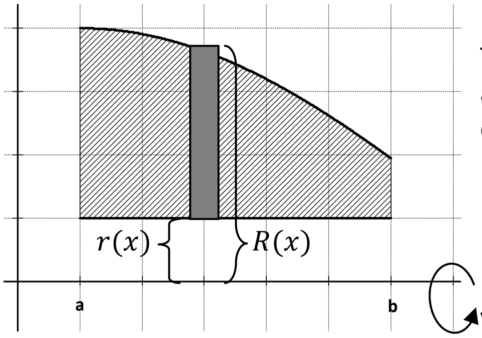
Date:________________

Unit 6: Applications of Integration

6.5 Volume – Washer Method

Extension of disc method to cover solids of revolution with holes. [applet1][applet2] [applet3] [link] [applet4]

*results from rotating a region that is not always touching the axis of revolution.



The representative rectangle is still perpendiular to, but will not always touch, the axis of revolution. This causes two radii to emerge, one from the axis of revolution to the outside of the region ($R(x) = outer\ radius$) and one from the axis of revolution to the inside of the region ($r(x) = inner\ radius$).

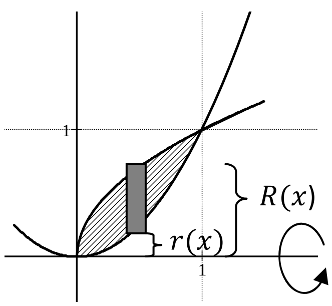
*use the disc method to subtract the hole volume from the total volume = washer method

$$\boxed{V = \pi \int_a^b ([R(x)]^2 - [r(x)]^2)\, dx}$$

similarly if rotating about a vertical axis of revolution:

$$\boxed{V = \pi \int_a^b ([R(y)]^2 - [r(y)]^2)\, dy}$$

ex. Find the volume of the solid generated rotating the region bounded by $y = \sqrt{x}$ & $y = x^2$ about the x-axis.



intersections: $\sqrt{x} = x^2$

$$x = x^4$$
$$x^4 - x = 0$$
$$x(x^3 - 1) = 0$$
$$x = 0 \text{ or } x = 1$$

$R(x) = \sqrt{x}$

$r(x) = x^2$

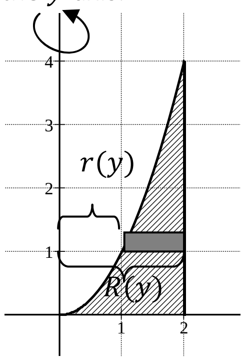
$$V = \pi \int_0^1 \left(\left[\sqrt{x}\right]^2 - [x^2]^2\right) dx$$

$$= \pi \int_0^1 x - x^4 dx$$

$$= \pi \left[\frac{x^2}{2} - \frac{x^5}{5}\right]_0^1$$

$$\boxed{= \frac{3\pi}{10}\ un^3}$$

ex. Find the volume of the solid generated rotating the region bounded by $y = x^2, y = 0$ & $x = 2$ about the y-axis.



*note: if rotated about x-axis it is a solid object and the disc method is used

$y = x^2 \rightarrow x = \sqrt{y}$

$R(y) = 2$

$r(y) = \sqrt{y}$

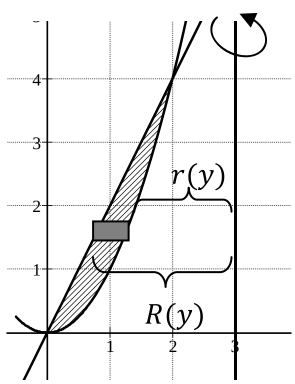
$$V = \pi \int_0^4 \left([2]^2 - \left[\sqrt{y}\right]^2\right) dy$$

$$= \pi \int_0^4 4 - y\, dy$$

$$= \pi \left[4y - \frac{y^2}{2}\right]_0^4$$

$$\boxed{= 8\pi\ un^3}$$

ex. Find the volume of the solid generated rotating the region bounded by $y = x^2$ & $y = 2x$ about $x = 3$. Solve using the integral feature of your calculator.



vertical axis:

$y = x^2 \rightarrow x = \sqrt{y}$

$y = 2x \rightarrow x = \frac{y}{2}$

$R(y) = 3 - \frac{y}{2}$

$r(y) = 3 - \sqrt{y}$

intercepts:

$\sqrt{y} = \frac{y}{2}$

$y = \frac{y^2}{4}$

$y^2 - 4y = 0$

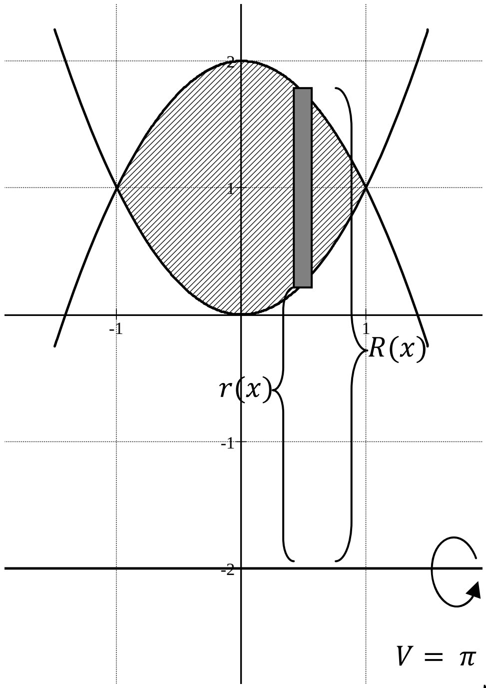
$y(y - 4) = 0$

$y = 0\ \ or\ \ y = 4$

$$V = \pi \int_0^4 \left(\left[3 - \frac{y}{2}\right]^2 - \left[3 - \sqrt{y}\right]^2\right) dy$$

$$\boxed{\cong 16.8\ un^3}$$

ex. Find the volume of the solid generated rotating the region bounded by $y = x^2$ & $y = 2 - x^2$ about $y = -2$. Solve using the integral feature of your calculator.



intersections:

$$x^2 = 2 - x^2$$
$$2x^2 = 2$$
$$x^2 = 1$$
$$x = \pm 1$$

$R(x) = 2 - x^2 + 2 = 4 - x^2$

$r(x) = x^2 + 2$

$$V = \pi \int_{-1}^{1} ([4 - x^2]^2 - [x^2 + 2]^2)\, dx$$

or (by symmetry):

$$V = 2\pi \int_{0}^{1} ([4 - x^2]^2 - [x^2 + 2]^2)\, dx$$

$$\boxed{= 16\pi\ un^3}$$

Practice Questions

pg. 423/ #5, 6, 11bd, 12ac, 13 – 15, 17, 20, 21, 30