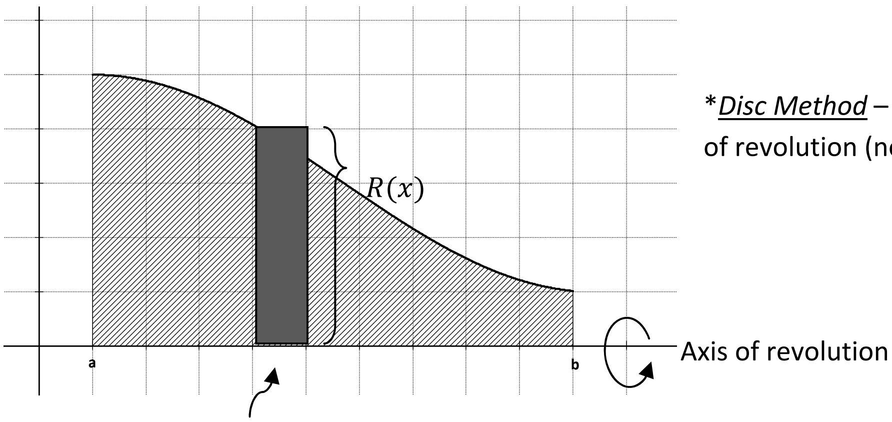
Unit 6: Applications of Integration
6.4 Volume – Disc Method

When a region in the plane is revolved about a line (*axis of revolution*) it produces a *solid of revolution* [applet][link][applet2].

Area under a curve – limit of sum of rectangle areas

Volume of Revolution – limit of sum of disc volumes



*<u>*Disc Method*</u> – entire region is flush to the axis of revolution (no space between the two).

representative rectangle – perpendicular to the axis of revolution with height $R(x)$

*rotating the rectangle creates a disc of radius R and width Δx

$$Volume\ of\ disc = \pi R^2 \Delta x$$

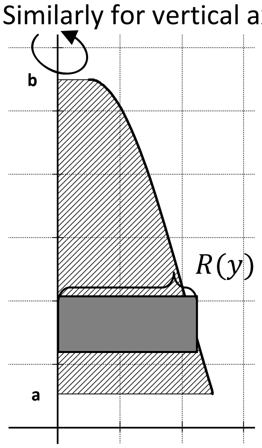
We can approximate the volume of the solid by adding the volume of n discs and then determine the actual volume by limiting the number of discs to infinity. (Similar to how we limited Riemann sums to infinity to find area under a curve in unit 3.)

$$Volume = \lim_{n\to\infty} \pi \sum_{i=1}^{n} [R(x_i)]^2\, \Delta x$$

$$\boxed{\therefore V = \pi \int_a^b [R(x)]^2\, dx}$$

Volume of a solid of revolution with horizontal axis of revolution where $R(x)$ is the function that gives the length of the representative rectangle.

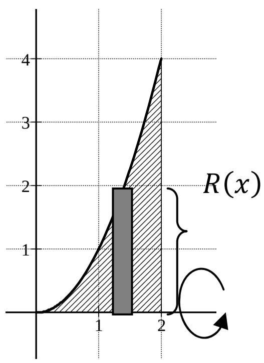
Similarly for vertical axis of revolution (with respect to y).



$$\boxed{\therefore V = \pi \int_a^b [R(y)]^2\, dy}$$

ex. Find the volume of the solid generated rotating the region bounded by $y = x^2, y = 0, x = 0$ & $x = 2$ about the x-axis.

*draw a sketch:



$$R(x) = x^2$$

$$\therefore V = \pi \int_0^2 (x^2)^2 \, dx$$

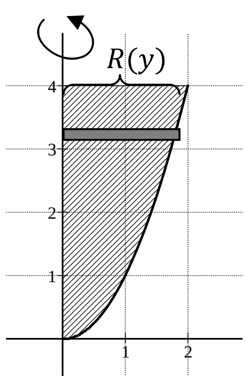
$$= \pi \int_0^2 x^4 \, dx$$

$$= \pi \left[\frac{x^5}{5}\right]_0^2$$

$$\boxed{= \frac{32\pi}{5} \, un^3}$$

*check with calc

ex. Find the volume of the solid generated rotating the region bounded by $y = x^2, x = 0$ & $y = 4$ about the y-axis.



$$y = x^2 \rightarrow x = \sqrt{y}$$

$$R(y) = \sqrt{y}$$

$$\therefore V = \pi \int_0^4 \left(\sqrt{y}\right)^2 dy$$

$$= \pi \int_0^4 y \, dy$$

$$= \pi \left[\frac{y^2}{2}\right]_0^4$$

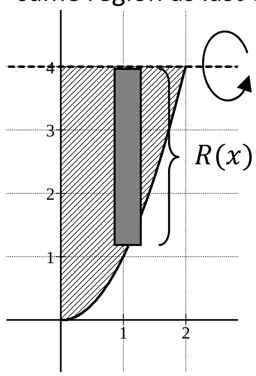
$$\boxed{= 8\pi \, un^3}$$

*check with calc

Can also rotate about lines that are not the grid axis.

ex. Find the volume of the solid generated rotating the region bounded by $y = x^2, x = 0$ & $y = 4$ about $y = 4$.

*same region as last example but now rotated horizontally



$R(x) = 4 - x^2$

$$\therefore V = \pi \int_0^2 (4 - x^2)^2 \, dx$$

$$= \pi \int_0^2 16 - 8x^2 + x^4 \, dx$$

$$= \pi \left[16x - \frac{8x^3}{3} + \frac{x^5}{5} \right]_0^2$$

$$\boxed{= \frac{256\pi}{15} \, un^3}$$

*check with calc

Practice Questions

pg. 423/ #1 – 4, 7 – 10, 11ac, 12bd, 16, 19, 22 – 29, 31 – 36 (use calc to solve integrals), 41, 42