Calc12 Date:________________

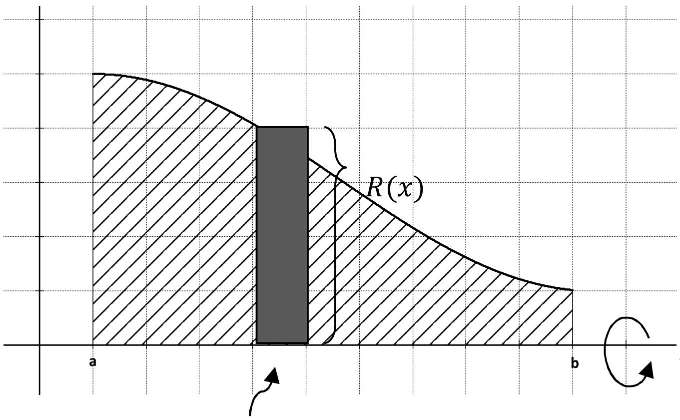
Mr. C. Sdoutz

Unit 6: Applications of Integration
6.4 Volume – Disc Method

When a region in the plane is revolved about a line (*axis of revolution*) it produces a *solid of revolution* [applet][link][applet2].

Area under a curve – limit of sum of rectangle areas

Volume of Revolution – limit of sum of disc volumes



**Disc Method* – entire region is flush to the axis of revolution (no space between the two).

representative rectangle – perpendicular to the axis of revolution with height $R(x)$

*rotating the rectangle creates a disc of radius R and width Δx

$$Volume\ of\ disc = \pi R^2 \Delta x$$

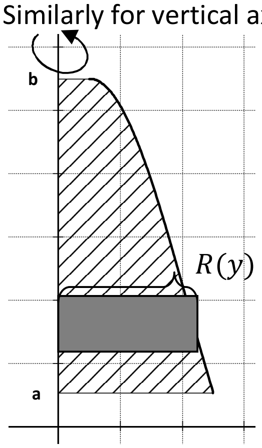
We can approximate the volume of the solid by adding the volume of n discs and then determine the actual volume by limiting the number of discs to infinity. (Similar to how we limited Riemann sums to infinity to find area under a curve in unit 3.)

$$Volume = \lim_{n\to\infty} \pi \sum_{i=1}^{n} [R(x_i)]^2 \, \Delta x$$

$$\boxed{\therefore V = \pi \int_a^b [R(x)]^2 \, dx}$$

Volume of a solid of revolution with horizontal axis of revolution where $R(x)$ is the function that gives the length of the representative rectangle.

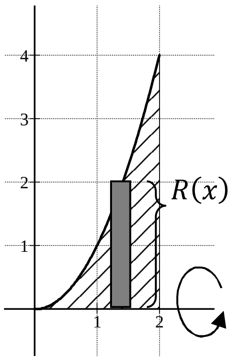
Similarly for vertical axis of revolution (with respect to y).



$$\boxed{\therefore V = \pi \int_a^b [R(y)]^2 \, dy}$$

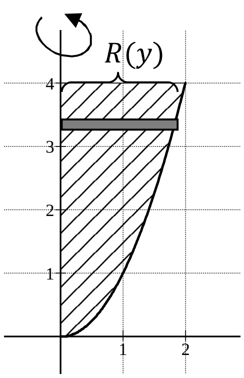
ex. Find the volume of the solid generated rotating the region bounded by $y = x^2, y = 0, x = 0$ & $x = 2$ about the x-axis.

*draw a sketch:



$R(x) =$

ex. Find the volume of the solid generated rotating the region bounded by $y = x^2, x = 0$ & $y = 4$ about the y-axis.



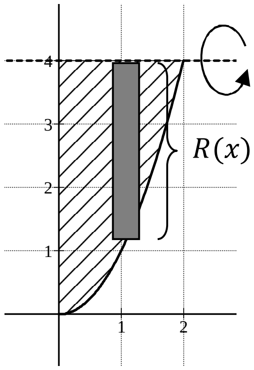
$y = x^2 \rightarrow$

$R(y) =$

Can also rotate about lines that are not the grid axis.

ex. Find the volume of the solid generated rotating the region bounded by $y = x^2, x = 0\ \& \ y = 4$ about $y = 4$.

*same region as last example but now rotated <u>horizontally</u>



$R(x) =$

Practice Questions

pg. 423/ #1 – 4, 7 – 10, 11ac, 12bd, 16, 19, 22 – 29, 31 – 36 (use calc to solve integrals), 41, 42