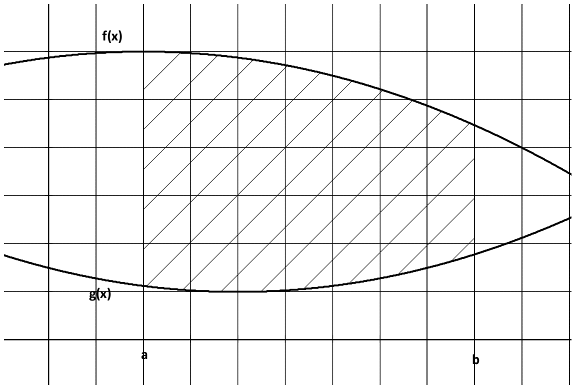
APCalc
Mr. C. Sdoutz

Date:________________

Unit 6: Applications of Integration
6.4 Area Between Curves



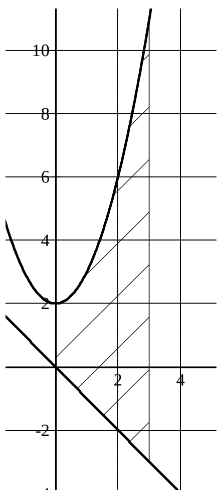
Area = area under f – area under g

$$A = \int_a^b f(x)\,dx - \int_a^b g(x)\,dx$$

$$\boxed{A = \int_a^b [f(x) - g(x)]\,dx}$$

*$f(x) \geq g(x)$ for all x in $[a, b]$

ex. Find the area of the region bounded by $y = x^2 + 2, y = -x, x = 0,$ and $x = 3.$



Which function is greater on the interval? – check with calc. or sub in a value in the interval.

$$A = \int_0^3 [(x^2 + 2) - (-x))dx$$

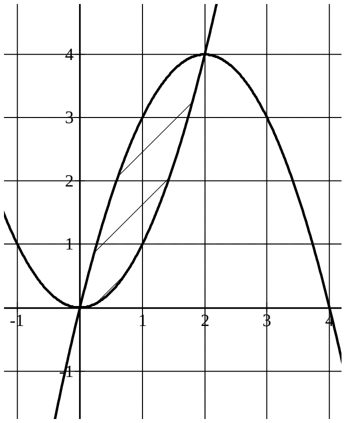
$$= \int_0^3 x^2 + x + 2dx$$

$$= \left[\frac{x^3}{3} + \frac{x^2}{2} + 2x\right]_0^3$$

$$= 9 + \frac{9}{2} + 6 - 0$$

$$\boxed{= \frac{39}{2} un^2}$$

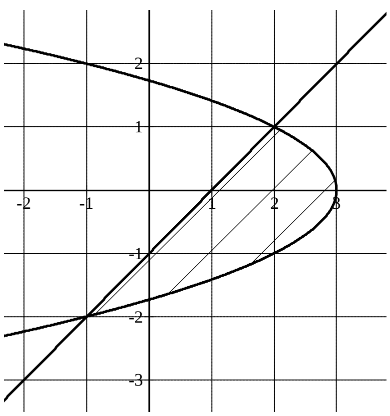
ex. Find the area between $y = x^2$ and $y = -x^2 + 4x$.
*no bounds given thus it is assumed the intersections of the functions are the bounds



$$x^2 = -x^2 + 4x$$
$$2x^2 - 4x = 0$$
$$2x(x-2) = 0$$
$$x = 0 \;\; or \; x = 2$$

$$A = \int_0^2 [(-x^2 + 4x) - (x^2)]\, dx$$
$$= \int_0^2 -2x^2 + 4x\, dx$$
$$= \left[\frac{-2x^3}{3} + 2x^2\right]_0^2$$
$$= -\frac{16}{3} + 8 - 0$$
$$\boxed{= \frac{8}{3} un^2}$$

ex. Find the area between $x = 3 - y^2$ and $y = x - 1$.



*There is no single region between the curves with respect to x (horizontally) but there is a single region between the curves with respect to y (vertically). Thus we will integrate with respect to y.

Rearrange both equations to $x =$ form:

$$x = 3 - y^2 \;\&\; x = y + 1$$

Find intersections:

$$3 - y^2 = y + 1$$
$$y^2 + y - 2 = 0$$
$$(y+2)(y-1) = 0$$
$$y = -2 \; or \; y = 1$$

Find area:

$$A = \int_{-2}^{1} [(3 - y^2) - (y + 1)]\, dy$$
$$= \int_{-2}^{1} -y^2 - y + 2dy$$
$$= \left[-\frac{y^3}{3} - \frac{y^2}{2} + 2y\right]_{-2}^{1}$$
$$= \left(-\frac{1}{3} - \frac{1}{2} + 2\right) - \left(\frac{8}{3} - 2 - 4\right)$$
$$\boxed{= \frac{9}{2} un^2}$$

Practice Questions

pg. 413/ #1 – 6, 13 – 23, 27 – 32