

Unit 5: Logarithmic, Exponential, & Inverse Trigonometric Functions
5.4 Bases Other Than e

Let b be a positive real number $\neq 1$, then:

$$\frac{d}{dx} [b^x] = b^x \ln b$$

$$\frac{d}{dx} [\log_b x] = \frac{1}{x \ln b}$$

$$\int b^x dx = \frac{b^x}{\ln b} + c$$

ex. Differentiate:

a) $y = 2^x$

$$\boxed{\frac{dy}{dx} = 2^x \ln 2}$$

b) $f(x) = 3^{x^2}$

$$\boxed{f'(x) = 3^{x^2} \ln 3 \cdot 2x}$$

c) $f(x) = \log_5 \sin x$

$$\begin{aligned} f'(x) &= \frac{1}{\sin x \ln 5} \cdot \cos x \\ &= \frac{\cos x}{\sin x \ln 5} \\ &= \boxed{\frac{\cot x}{\ln 5}} \end{aligned}$$

ex.

$$\int 2^x dx$$

$$\boxed{= \frac{2^x}{\ln 2} + c}$$

ex.

$$\int_1^e \frac{5^{\ln x}}{x} dx$$

let $u = \ln x$

then $\frac{du}{dx} = \frac{1}{x}$

$x = e \rightarrow u = \ln e = 1$

and $dx = x du$

$x = 1 \rightarrow u = \ln 1 = 0$

$$= \int_0^1 \frac{5^u}{x} x du$$

$$= \int_0^1 5^u du$$

$$= \left[\frac{5^u}{\ln 5} \right]_0^1$$

$$= \frac{1}{\ln 5} (5^1 - 5^0)$$

$$\boxed{= \frac{4}{\ln 5}}$$

Practice Questions

Pg. 354/ #29 – 34, 37 – 43, 69 – 71, 73 – 76