Unit 5: Logarithmic, Exponential, & Inverse Trigonometric Functions
5.3 The Natural Exponential Function

The natural exponential function $y = e^x$ is extremely unique in calculus because it is its own derivative and its own anti-derivative.
ie.

$$\frac{d}{dx}[e^x] = e^x \qquad \text{and} \qquad \int e^x\,dx = e^x + c$$

ex. Differentiate each function:

a) $f(x) = e^{x^2}$
*chain rule
$f'(x) = e^{x^2} \cdot 2x$
$\boxed{= 2xe^{x^2}}$

b) $f(x) = \frac{1}{e^{2x}}$
*rewrite
$f(x) = e^{-2x}$
then
$f'(x) = e^{-2x} \cdot -2$
$\boxed{= \frac{-2}{e^{2x}}}$

ex. Solve: *for integrals let $u =$ the exponent function

a)
$\int e^{3x}\,dx$
let $u = 3x$
then $\frac{du}{dx} = 3$
and $dx = \frac{du}{3}$
$= \int e^u \frac{du}{3}$
$= \frac{1}{3}e^u + c$
$\boxed{= \frac{e^{3x}}{3} + c}$

b)
$\int \sin x\, e^{\cos x} dx$
let $u = \cos x$
then $\frac{du}{dx} = -\sin x$
and $dx = \frac{du}{-\sin x}$
$= \int \sin x\, e^u \frac{du}{-\sin x}$
$= -\int e^u\,du$
$\boxed{= -e^{\cos x} + c}$

ex. Solve:

a)

$$\int_0^1 e^{-x}\,dx$$

let $u = -x$

then $\frac{du}{dx} = -1$

and $dx = -du$

$x = 1 \rightarrow u = -1$

$x = 0 \rightarrow u = 0$

$$= -\int_0^{-1} e^u\,du$$

$$= -[e^u]_0^{-1}$$

$$= -(e^{-1} - e^0)$$

$$\boxed{= \frac{-1}{e} + 1 \cong 0.632}$$

b)

$$\int_0^1 \frac{e^x}{e^x + 1}$$

let $u = e^x + 1$

then $\frac{du}{dx} = e^x$

and $dx = \frac{du}{e^x}$

$x = 1 \rightarrow u = e + 1$

$x = 0 \rightarrow u = 2$

$$= \int_2^{1+e} \frac{e^x}{u}\frac{du}{e^x}$$

$$= [\ln|u|]_2^{1+e}$$

$$\boxed{\begin{aligned} &= \ln(1+e) - \ln 2 \\ &\cong 0.620 \end{aligned}}$$

Practice Questions

Pg. 344/ #27 – 39, 43 – 52, 58, 59, 76 – 83, 87, 88, 92, 99, 101, 102