Unit 5: Logarithmic, Exponential, & Inverse Trigonometric Functions
5.2 Natural Log & Integration

Since

$$\frac{d}{dx}[\ln x] = \frac{1}{x}$$

Then

$$\boxed{\int \frac{1}{x}\,dx = \ln|x| + c}$$

*need absolute value since we can only log positive numbers

note: the power integration law $\int x^n\,dx = \frac{x^{n+1}}{n+1} + c$ does not work on $\frac{1}{x}$

$$\int \frac{1}{x}\,dx = \int x^{-1}\,dx = \frac{x^0}{0} + c$$

ex.

$$\int \frac{2}{x}\,dx$$

$$= 2\int \frac{1}{x}\,dx$$

$$= 2\ln|x| + c$$

$$\boxed{= \ln x^2 + c}$$

* x^2 always positive so can drop absolute value

ex.

$$\int \frac{x^2+1}{x^3+3x}\,dx$$

*integration of rational functions could lead to a natural log antiderivative, thus we use substitution method (let $u =$ the denominator)

let $u = x^3 + 3x$
then $\frac{du}{dx} = 3x^2 + 3$
and $dx = \frac{du}{3(x^2+1)}$

$$= \int \frac{x^2+1}{u}\frac{du}{3(x^2+1)}$$

$$= \frac{1}{3}\int \frac{1}{u}\,du$$

$$= \frac{1}{3}\ln|u| + c$$

$$\boxed{= \frac{1}{3}\ln|x^3+3x| + c}$$

ex.

$\int \tan x \, dx$

rewrite:

$$= \int \frac{\sin x}{\cos x}$$

let $u = \cos x$

then $\frac{du}{dx} = -\sin x$

and $dx = \frac{du}{-\sin x}$

$$= \int \frac{\sin x}{u} \frac{du}{-\sin x}$$

$$= -\int \frac{1}{u} du$$

$$= -\ln|\cos x| + c$$

$$\boxed{= \ln|\sec x| + c}$$

ex. Determine the area under $y = \frac{x}{x^2+1}$ on $[0, 3]$.

$$\int_0^3 \frac{x}{x^2+1} dx$$

let $u = x^2 + 1$

then $\frac{du}{dx} = 2x$ $\qquad x = 3 \rightarrow u = 10$

and $dx = \frac{du}{2x}$ $\qquad x = 0 \rightarrow u = 1$

$$= \int_1^{10} \frac{x}{u} \frac{du}{2x}$$

$$= \frac{1}{2} \int_1^{10} \frac{1}{u} du$$

$$= \frac{1}{2} [\ln|u|]_1^{10}$$

$$= \frac{1}{2} (\ln 10 - \ln 1)$$

$$\boxed{= \frac{\ln 10}{2} \; un^2 \cong 1.151 \; un^2}$$

If the degree of the numerator is equal or greater than the degree of the denominator try long division to split up the integrand.

$$\frac{dividend}{divisor} = quotient + \frac{remainder}{divisor}$$

ex.

$$\int \frac{x^2 + x + 1}{x^2 + 1}$$

$$\begin{array}{r|l} & \quad\quad\quad\; 1 \\ \hline x^2+1 & x^2 + x + 1 \\ & \underline{x^2 \quad\;\; + 1} \\ & \quad\; x \end{array}$$

$$= \int 1 + \frac{x}{x^2 + 1}\, dx$$

$$= \int 1\, dx + \int \frac{x}{x^2+1} dx$$

$$= x + \int \frac{x}{x^2+1} dx$$

let $u = x^2 + 1$

then $\frac{du}{dx} = 2x$

and $dx = \frac{du}{2x}$

$$= x + \int \frac{x}{u}\frac{du}{2x}$$

$$= x + \frac{1}{2}\int \frac{1}{u} du$$

$$= x + \frac{1}{2}\ln|u| + c$$

$$\boxed{= x + \frac{1}{2}\ln(x^2+1) + c = x + \ln\sqrt{x^2+1} + c}$$

Practice Questions

Pg. 327/# 1 – 14, 19, 20, 23, 24, 33 – 38, 51 – 54, 57, 58, 61, 63, 69, 70