Unit 5: Logarithmic, Exponential, & Inverse Trigonometric Functions
5.1 Natural Log & Differentiation

recall:

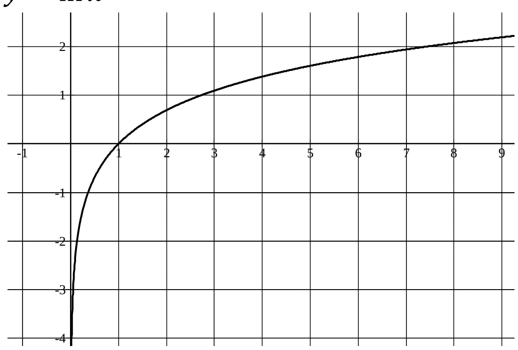
$$\log_b x = y \quad \leftrightarrow \quad b^y = x$$

ex. $\log_2 8 = 3$

Common logarithms are base 10 logarithms and written without the base: $\log 100 = 2$

Natural logarithms are base e logarithms ($e \cong 2.71828\ 18284\ 59045\ 23536\ \ldots.$) and are denoted ln:
ie. $\log_e x = \ln x$
thus $\ln x = y \quad \leftrightarrow \quad e^y = x$

graphically like all log functions:
ex.
$y = \ln x$



Domain: $x > 0$

Range: $\mathbb{R}$

Same properties as logarithms:
1. $\ln 1 = 0$
2. $\ln(ab) = \ln a + \ln b$
3. $\ln\frac{a}{b} = \ln a - \ln b$
4. $\ln a^n = n \ln a$

ex. $\ln\frac{x^2(x-2)}{x+5} = 2\ln x + \ln(x-2) - \ln(x+5)$

Differentiation

$$\boxed{\frac{d}{dx}[\ln x] = \frac{1}{x}\,;x > 0}$$

ex.

$$\frac{d}{dx}[\ln 2x] = \frac{1}{2x} \cdot 2 = \boxed{\frac{1}{x}}$$

chain rule!

ex. $f(x) = \ln(x^2+1)$

$$f'(x) = \frac{1}{x^2+1} \cdot 2x$$

$$\boxed{= \frac{2x}{x^2+1}}$$

ex. $y = x \ln x$ *product rule!

$$\frac{dy}{dx} = x \cdot \frac{1}{x} + \ln x \cdot 1$$

$$\boxed{= 1 + \ln x}$$

ex. $f(x) = (\ln x)^3$ *chain rule!

$$f'(x) = 3(\ln x)^2 \cdot \frac{1}{x}$$

$$\boxed{= \frac{3(\ln x)^2}{x}}$$

ex.
$\frac{d}{dx}[\ln|\cos x|]$

$$= \frac{1}{\cos x} \cdot -\sin x$$

$$\boxed{= -\tan x}$$

Use logarithm properties to split up complicated log functions to aid the differentiation.

ex. $f(x) = \ln\sqrt{x+1}$

rewrite: $f(x) = \ln(x+1)^{1/2} = \frac{1}{2}\ln(x+1)$

$$f'(x) = \frac{1}{2} \cdot \frac{1}{x+1} \cdot 1$$

$$\boxed{= \frac{1}{2(x+1)}}$$

ex. $f(x) = \ln\frac{x(x^2+1)^2}{\sqrt{2x^3-1}}$

$$f(x) = \ln x + 2\ln(x^2+1) - \frac{1}{2}\ln(2x^3-1)$$

$$f'(x) = \frac{1}{x} + 2 \cdot \frac{1}{x^2+1} \cdot 2x - \frac{1}{2} \cdot \frac{1}{2x^3-1} \cdot 6x^2$$

$$\boxed{= \frac{1}{x} + \frac{4x}{x^2+1} - \frac{3x^2}{2x^3-1}}$$

Practice Questions

Pg. 318/# 13 – 30, 37 – 55, 67 – 70, 73 – 78