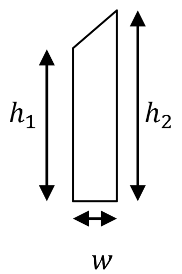
APCalc Date:________________
Mr. C. Sdoutz

Unit 4: Integration
4.8 Trapezoidal Rule

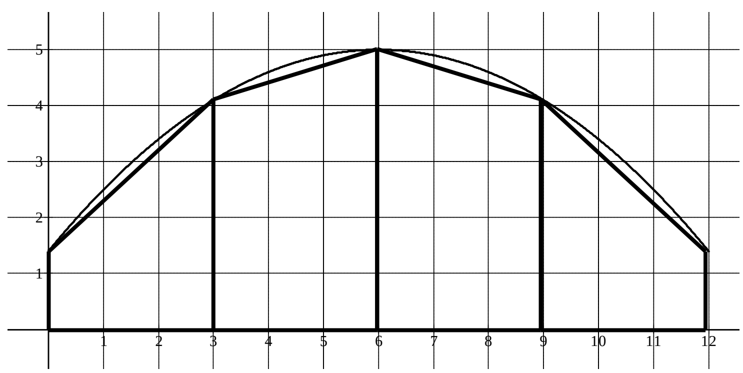
An alternate method (and more accurate) to using rectangles as an approximation to area (and integrals) is to use trapezoids.



Area $= \left(\frac{h_1+h_2}{2}\right) w$

$*w = \Delta x = \frac{b-a}{n}$

ex. Use 4 trapezoids to approximate $\int_0^{12} f(x)\, dx$:



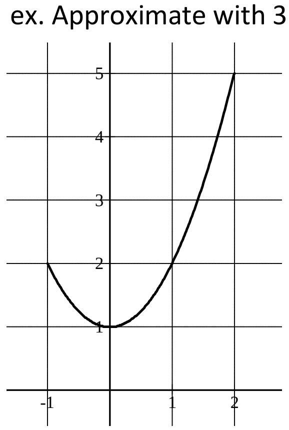
$\Delta x = \frac{12-0}{4} = 3$

$$\cong 3\left(\frac{f(0)+f(3)}{2}\right) + 3\left(\frac{f(3)+f(6)}{2}\right) + 3\left(\frac{f(6)+f(9)}{2}\right) + 3\left(\frac{f(9)+f(12)}{2}\right)$$
$$\cong \frac{3}{2}[f(0) + 2f(3) + 2f(6) + 2f(9) + f(12)]$$

Trapezoidal Approximation with n rectangles:
$$\int_a^b f(x)\, dx = \frac{b-a}{2n}[f(x_0) + 2f(x_1) + 2f(x_2) + \cdots + 2f(x_{n-1}) + f(x_n)]$$

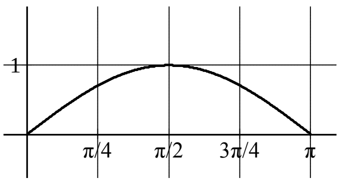
*note: the average of the upper & lower sums

ex. Approximate with 3 trapezoids:



$$\int_{-1}^{2} (x^2+1)\, dx$$
$$\cong \frac{2-(-1)}{2(3)}[f(-1) + 2f(0) + 2f(1) + f(2)]$$
$$\cong \frac{1}{2}[2 + 2(1) + 2(2) + 5]$$
$$\boxed{\cong \frac{13}{2}}$$

ex. Approximate with 4 trapezoids:



$$\int_0^{\pi} \sin x \, dx$$

$$\cong \frac{\pi - 0}{2(4)}\left[\sin 0 + 2\sin\frac{\pi}{4} + 2\sin\frac{\pi}{2} + 2\sin\frac{3\pi}{4} + \sin\pi\right]$$

$$\cong \frac{\pi}{8}\left[0 + 2\left(\frac{1}{\sqrt{2}}\right) + 2(1) + 2\left(\frac{1}{\sqrt{2}}\right) + 0\right]$$

$$\cong \frac{\pi}{8}\left(\frac{4}{\sqrt{2}} + 2\right)$$

$$\cong \frac{\pi}{8}\left(2\sqrt{2} + 2\right)$$

$$\boxed{\cong \frac{\pi\left(\sqrt{2} + 1\right)}{4}}$$

Practice Questions

Pg. 304/# 2 – 14 even (trapezoid rule <u>only</u>), 40a