APCalc Date:_____
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Unit 4: Integration
4.7 Integration By Substitution

*used to integrate composite functions
ex.

$$\int x^2(2x^3+2)^3\,dx$$

let $u = 2x^3 + 2$ *substitute for the inner function
then $\frac{du}{dx} = 6x^2$ *differentiate and solve for dx
$dx = \frac{du}{6x^2}$ *now substitute back into the integral

$$= \int x^2 \cdot u^3 \frac{du}{6x^2}$$
$$= \frac{1}{6}\int u^3\,du$$
$$= \frac{1}{6}\left(\frac{u^4}{4}\right) + c$$
$$= \frac{u^4}{24} + c$$
$$\boxed{= \frac{(2x^3+2)^4}{24} + c}$$

ex.

$$\int x^3\sqrt{1+x^2}\,dx$$

let $u = 1 + x^2$
then $\frac{du}{dx} = 2x$
$dx = \frac{du}{2x}$

$$= \int x^3\sqrt{u}\frac{du}{2x}$$
$$= \frac{1}{2}\int x^2\sqrt{u}\,du$$

*still need to eliminate x
since $u = 1 + x^2$
then $x^2 = u - 1$

$$= \frac{1}{2}\int (u-1)u^{1/2}\,du$$
$$= \frac{1}{2}\int \left(u^{3/2} - u^{1/2}\right)du$$
$$= \frac{1}{2}\left(\frac{2}{5}u^{5/2} - \frac{2}{3}u^{3/2}\right) + c$$
$$\boxed{= \frac{1}{5}(1+x^2)^{5/2} - \frac{1}{3}(1+x^2)^{3/2} + c}$$

Extend to definite integrals as well:

ex.

$$\int_0^1 x(x^2+1)^3\,dx$$

let $u = x^2 + 1$ also switch the values of the bounds:

then $\frac{du}{dx} = 2x$ $x = 1 \Rightarrow u = 2$

$dx = \frac{du}{2x}$ $x = 0 \Rightarrow u = 1$

$$= \int_1^2 x \cdot u^3 \frac{du}{2x}$$

$$= \frac{1}{2}\int_1^2 u^3\,du$$

$$= \frac{1}{2}\left[\frac{u^4}{4}\right]_1^2$$

$$= \frac{1}{2}\left(4 - \frac{1}{4}\right)$$

$$\boxed{= \frac{15}{8}}$$

ex.

$$\int_0^{\pi/2} \sin^2 x\ cosx\,dx$$

let $u = \sin x$

then $\frac{du}{dx} = \cos x$ $x = \pi/2 \Rightarrow u = 1$

$dx = \frac{du}{cosx}$ $x = 0 \Rightarrow u = 0$

$$= \int_0^1 u^2\ cosx \frac{du}{cosx}$$

$$= \int_0^1 u^2\,du$$

$$= \left[\frac{u^3}{3}\right]_0^1$$

$$\boxed{= \frac{1}{3}}$$

Practice Questions

Pg. 296/# 7 – 17, 27, 29, 33 – 36, 45, 47 – 50, 54 – 58, 60 – 62, 65 – 67