Calc12 Date:________________

Mr. C. Sdoutz

Unit 4: Integration
4.6 Mean Value Theorem & The 2nd FTC

Theorem

If f is continuous on $[a, b]$, then there exists a value c in $[a, b]$ such that

$$\int_a^b f(x)\,dx = f(c)(b-a)$$

Geometric interpretation: there is a rectangle with height $f(c)$ and width $(b-a)$ which has area equal to the actual area under the curve.

The value of $f(c)$ in the theorem is called the *average value* of f on $[a, b]$.
*not the same as average rate of change of f

Definition

The average value of f on $[a, b]$ is

$$f(c) = \frac{1}{b-a}\int_a^b f(x)\,dx$$

ex. Find the average value of $f(x) = 3x^2 - 2x$ on $[1, 4]$ and determine the value c in the interval where the average value occurs.

If the upper bound of a definite integral is a variable, instead of a number, then the definite integral will be a function.

ex.

$$\int_0^x \cos t \, dt$$

ex.

$$\int_2^x 2t \, dt$$

If we take the derivative of one of these integral functions note how the answer becomes the intgrand with a change of variable.

ex.

$$\frac{d}{dx}\int_0^x \cos t \, dt$$

ex.

$$\frac{d}{dx}\int_2^x 2t \, dt$$

*note: the value of the lower limit is irrelevant

<u>The 2nd Fundamental Theorem of Calculus</u>

If f is continuous on an open interval containing a value c, then for every x in the interval

$$\frac{d}{dx}\int_c^x f(t)\,dt = f(x) \quad \text{and} \quad \frac{d}{dx}\int_c^{g(x)} f(t)\,dt = f(g(x)) \cdot\ g'(x)$$

ex.

If $F(x) = \int_3^x 3t^2 + 2\,dt$, find $F'(x)$

ex. Evaluate

$$\frac{d}{dx}\int_{\pi/2}^{x^3} \cos t\,dt$$

note: The constant **must** be the lower limit of integration.

ex. Evaluate

$$\frac{d}{dx}\int_{3x^2}^{5} 3 - 2t\,dt$$

Practice Questions

Pg. 284/# 43, 44, 46 – 50, 56, 57, 60, 66 – 70, 73 – 84, 89 – 91