APCalc
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Date:________________

Unit 4: Integration
4.5 Fundamental Theorem of Calculus

The Fundamental Theorem of Calculus
If a function f is continuous on $[a, b]$ and F is an antiderivative of f (ie. $F' = f$) on $[a, b]$ then

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

ie. Use the antiderivative of f to evaluate the definite integral instead of using the Riemann sum limit.

ex.

$\int_1^3 3x^2\,dx$

*recall (4.1) that $\int 3x^2\,dx = x^3 + c$

$= [x^3]_1^3$ *note the notation & constant not needed

$= (3)^3 - (1)^3$

$= 27 - 1$

$\boxed{= 26}$

ex.

$\int_1^2 (x^2 - 3)\,dx$

$= \left[\frac{x^3}{3} - 3x\right]_1^2$

$= \left(\frac{(2)^3}{3} - 3(2)\right) - \left(\frac{(1)^3}{3} - 3(1)\right)$

$= \frac{8}{3} - 6 - \frac{1}{3} + 3$

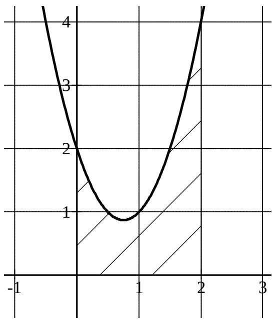
$= \frac{7}{3} - 3$

$= \frac{7}{3} - \frac{9}{3}$

$\boxed{= -\frac{2}{3}}$

Now we can also find the area under a curve much simpler.

ex. Find the area of the region bounded by $f(x) = 2x^2 - 3x + 2$, $y = 0$ (ie. the x-axis), $x = 0$ and $x = 2$.



f is non-negative on $[0, 2]$, thus

$$\begin{aligned}
Area &= \int_0^2 2x^2 - 3x + 2\,dx \\
&= \left[\frac{2x^3}{3} - \frac{3x^2}{2} + 2x\right]_0^2 \\
&= \left(\frac{2(2)^3}{3} - \frac{3(2)^2}{2} + 2(2)\right) - \left(\frac{2(0)^3}{3} - \frac{3(0)^2}{2} + 2(0)\right) \\
&= \frac{16}{3} - 6 + 4 - 0 \\
&= \frac{16}{3} - \frac{6}{3} \\
&\boxed{= \frac{10}{3} un^2}
\end{aligned}$$

Practice Questions

Pg. 283/# 5 – 20, 25, 30 – 42, 51 – 55