Date:________________

Unit 4: Integration
4.4 Definite Integrals

Definition

If f is defined on the closed interval $[a, b]$ and the limit

$$\lim_{n\to\infty}\sum_{i=1}^{n} f(c_i)\Delta x$$

exists, then f is *integrable* on $[a, b]$ and the limit is denoted by

$$\lim_{n\to\infty}\sum_{i=1}^{n} f(c_i)\Delta x = \int_a^b f(x)dx$$

This limit is called the *definite integral* of f from a to b. The number a is the lower limit of integration and the number b is the upper limit of integration.

The integral is a "net accumulator" – adds up terms like sigma.

Note: A definite integral is a number, while an indefinite integral is a group of functions.

Theorem

If f is continuous on $[a, b]$ then f is integrable on $[a, b]$.

ex. Evaluate:

$$\int_{-2}^{1} 2x\,dx = \lim_{n\to\infty}\sum_{i=1}^{n} f(c_i)\Delta x$$

$\Delta x = \frac{1-(-2)}{n} = \frac{3}{n}$ choose c_i to be right endpts $= a + i\Delta x = -2 + \frac{3i}{n}$

$$= \lim_{n\to\infty}\sum_{i=1}^{n} 2\left(-2+\frac{3i}{n}\right)\left(\frac{3}{n}\right)$$

$$= \lim_{n\to\infty}\frac{6}{n}\left[\sum_{i=1}^{n} -2 + \frac{3}{n}\sum_{i=1}^{n} i\right]$$

$$= \lim_{n\to\infty}\frac{6}{n}\left[-2n + \frac{3}{n}\left(\frac{n(n+1)}{2}\right)\right]$$

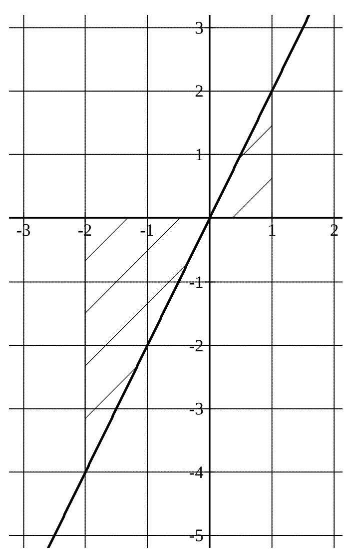
$$= \lim_{n\to\infty} 6\left[-2 + \frac{3}{n}\left(\frac{n+1}{2}\right)\right]$$

$$= \lim_{n\to\infty} 6\left[-2 + \frac{3n+3}{2n}\right]$$

$$= \lim_{n\to\infty} 6\left[\frac{-n+3}{2n}\right]$$

$$= \lim_{n\to\infty}\frac{-3n+9}{n}$$

$$\boxed{= -3}$$

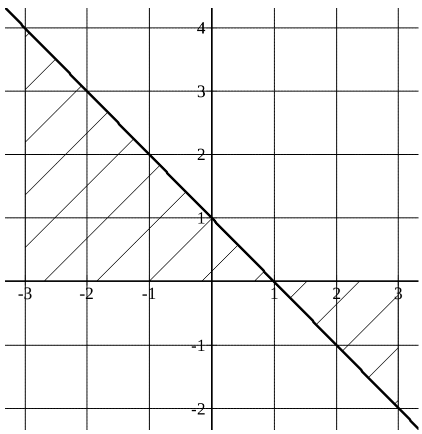


Although we use the limit of a Riemann sum to define the definite integral (like we did to find area in 4.3), the definite integral is a mathematical operation whose value can be positive or negative (as seen in the previous example). Using the definite integral to find area is only one application.
*any region below the x-axis has a negative integral value

ie. Area $= \int_a^b f(x)dx$ only if f is non-negative on $[a, b]$

We can still utilize the geometric interpretation of the definite integral (area) to find in definite integrals.

ex. Evaluate using a sketch and geometric formula:



$$\int_{-3}^{3} -x + 1\,dx$$
$$= \int_{-3}^{1} -x + 1\,dx + \int_{1}^{3} -x + 1\,dx$$
$$= \frac{(4)(4)}{2} - \frac{(2)(2)}{2}$$
$$= 8 - 2$$
$$\boxed{= 6}$$

Definite Integral Properties

$$\int_a^a f(x)\,dx = 0$$

$$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$$

$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$$

$$\int_a^b kf(x)\,dx = k\int_a^b f(x)\,dx$$

$$\int_a^b [f(x) \pm g(x)]\,dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$$

Practice Questions

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