Calc12 Date:________________
Mr. C. Sdoutz

Unit 4: Integration
4.4 Definite Integrals

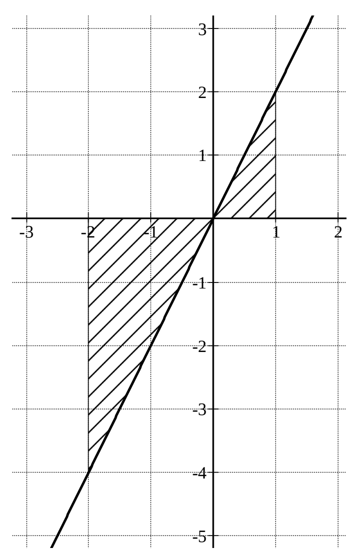
Definition
If f is defined on the closed interval $[a, b]$ and the limit
$$\lim_{n\to\infty}\sum_{i=1}^{n} f(c_i)\Delta x$$
exists, then f is *integrable* on $[a, b]$ and the limit is denoted by
$$\lim_{n\to\infty}\sum_{i=1}^{n} f(c_i)\Delta x = \int_a^b f(x)dx$$
This limit is called the *definite integral* of f from a to b. The number a is the lower limit of integration and the number b is the upper limit of integration.

The integral is a "net accumulator" – adds up terms like sigma.

Note: A definite integral is a number, while an indefinite integral is a group of functions.

Theorem
If f is continuous on $[a, b]$ then f is integrable on $[a, b]$.

ex. Evaluate:
$$\int_{-2}^{1} 2x\, dx$$

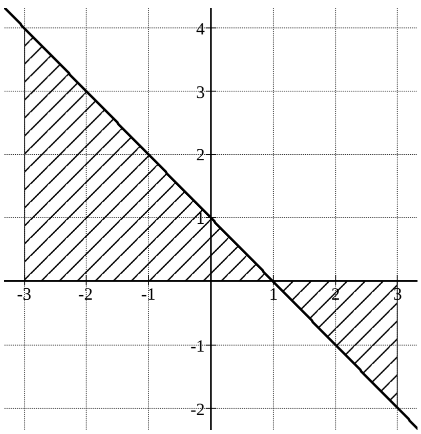


Although we use the limit of a Riemann sum to define the definite integral (like we did to find area in 4.3), the definite integral is a mathematical operation whose value can be positive or negative (as seen in the previous example). Using the definite integral to find area is only one application.
*any region below the x-axis has a negative integral value

ie. Area $= \int_a^b f(x)dx$ only if f is non-negative on $[a, b]$

We can still utilize the geometric interpretation of the definite integral (area) to find in definite integrals.

ex. Evaluate using a sketch and geometric formula:



$$\int_{-3}^{3} -x + 1\, dx$$

Definite Integral Properties

$$\int_a^a f(x)\, dx = 0$$

$$\int_a^b f(x)\, dx = -\int_b^a f(x)\, dx$$

$$\int_a^b f(x)\, dx = \int_a^c f(x)\, dx + \int_c^b f(x)\, dx$$

$$\int_a^b kf(x)\, dx = k\int_a^b f(x)\, dx$$

$$\int_a^b [f(x) \pm g(x)]\, dx = \int_a^b f(x)\, dx + \int_a^b g(x)\, dx$$

Practice Questions

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