Calc12 Date:________________

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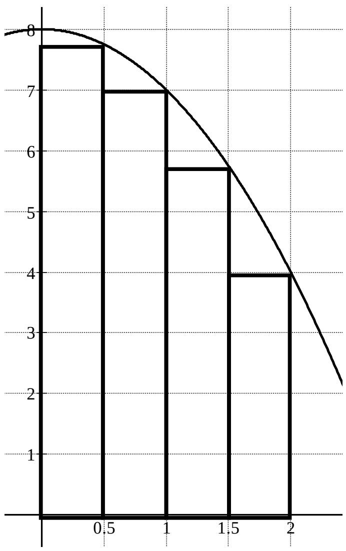
Unit 4: Integration
4.3 Riemann Sums

ex. Find a general expression for the lower and upper sums of $f(x) = -x^2 + 8$ on $[0, 2]$ using n rectangles.

Lowersum = right endpt rectangles *due to decreasing function



width of each rectangle:

$$\Delta x = \frac{b-a}{n} =$$

right endpts occur at:

$$a + i\Delta x =$$

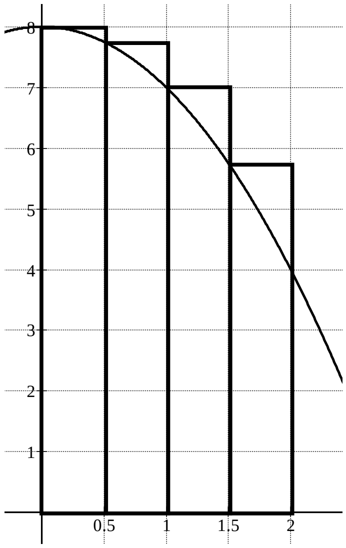
height of i^{th} rectangle:

Area of i^{th} rectangle:

$$A = wh =$$

Sum of n rectangles:

Uppersum = left endpt rectangles *due to decreasing function



width of each rectangle:

$$\Delta x = \frac{b-a}{n} =$$

left endpts occur at:

$$a + (i-1)\Delta x =$$

height of i^{th} rectangle:

Area of i^{th} rectangle:

$$A = wh =$$

Sum of n rectangles:

To find the *actual* area, take the limit as n approaches infinity:

Limit of lowersum: Limit of uppersum:

This is NOT a coincidence, to find the actual area it does not matter which endpts are used. Thus, use the **right endpts** as they are easier.

Riemann Sum Approximation

The area under $f(x)$ on $[a, b]$ using n rectangles.

Width of each rectangle $\Delta x = \frac{b-a}{n}$

Left endpts $a + (i-1)\Delta x$

Right endpts $a + i\Delta x$

$$Lowersum = s(n) = \sum_{i=1}^{n} f(m_i)\Delta x$$

$$Uppersum = S(n) = \sum_{i=1}^{n} f(M_i)\Delta x$$

[where m_i and M_i are the left or right endpts depending which give the desired sum]

Note: $\lim_{n\to\infty} s(n) = \lim_{n\to\infty} S(n)$

Definition of Area of a Region

Let f be continuous and non-negative on $[a, b]$, then the area of the region bounded by f, the x-axis, $x = a$, and $x = b$ is given by:

$$Area = \lim_{n\to\infty} \sum_{i=1}^{n} f(c_i)\Delta x$$

[where $\Delta x = \frac{b-a}{n}$ and c_i is an x-value in the i^{th} rectangle]

*We will let c_i be the right endpts.

ex. Find the area under $f(x) = x^2$ on $[2, 5]$

$\Delta x =$ right endpts $=$ height $=$

Thus, the Riemann sum approximation is:

Then, the area will be:

Practice Questions

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