Calc12 Date:________________

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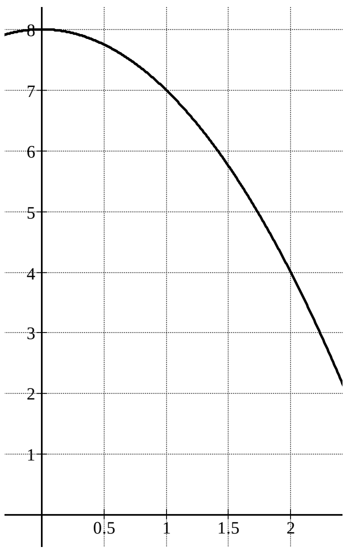
Unit 4: Integration
4.2 Area Approximation & Sigma Notation

Approximate the area under a curve using rectangles of equal width.

ex. Use four rectangles to approximate the area under $f(x) = -x^2 + 8$ on $[0, 2]$.

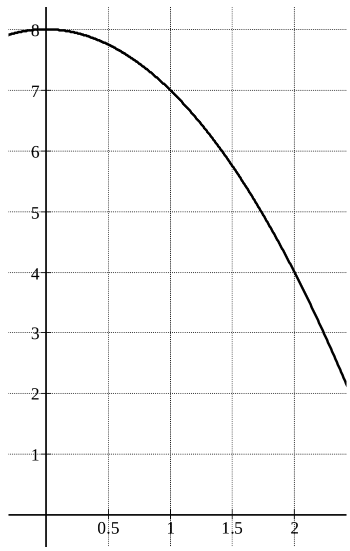
3 approaches

1) Left endpt. Rectangles



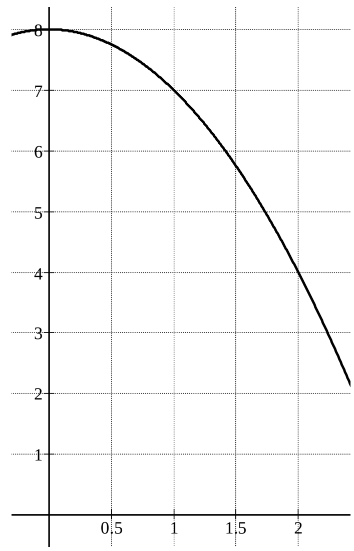
uppersum: area > actual

2) Midpt. Rectangles



midsum

3) Right endpt. Rectangles



lowersum: area < actual

width of each rectangle: $w =$

leftpts: $x =$

midpts: $x =$

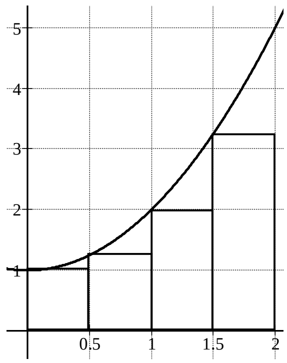
rightpts: $x =$

Area $=$ sum of the rectangle areas (ie. $A = \sum \text{rectangle areas}$)

Which is best?

How can we improve the approximation? [applet1][applet2]

*note: left endpts. does not always mean the uppersum (depends on if function is inc/dec)



ex. left endpts. = lowersum

Larger number of rectangles (thus leading to narrower width rectangles) will improve the approximation but becomes more tedious to calculate. Thus, we use ***sigma notation*** and ***summation formulas***.

We represent sums using the Greek letter Sigma Σ:

ex. "The sum from *k* equals one to five of two *k* minus three."

upper limit

term generator $\rightarrow t_n$ formula, generates the terms of the series

$$\sum_{k=1}^{5} 2k - 3$$

index of summation
*same variable in term generator

lower limit

* to find t_1 let the index variable = lower limit
to find t_2 let the index variable = lower limit + 1
to find t_3 let the index variable = lower limit + 2
etc...
to find last term let the index variable = upper limit

ex.

$$\sum_{i=1}^{5} i =$$

$$\sum_{j=3}^{6} j^2 =$$

Summation Properties

$$\sum_{i=1}^{n} ka_i = k\sum_{i=1}^{n} a_i \quad \text{where } k \text{ is a constant, and } a_i \text{ is the term generator}$$

ex.

$$\sum_{i=1}^{3} 2i^3 =$$

$$\sum_{i=1}^{n}(a_i \pm b_i) = \sum_{i=1}^{n} a_i \pm \sum_{i=1}^{n} b_i$$

ex.

$$\sum_{b=2}^{3}(b + b^2) =$$

Summation Formulas

$$\sum_{i=1}^{n} c = cn \text{ where } c \text{ is a constant}$$

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} = \frac{n(2n^2+3n+1)}{6}$$

$$\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$$

ex. evaluate for $n = 10$ and $n = 100$:

$$\sum_{i=1}^{n} \frac{i+1}{n^2}$$

Thus, if $n = 10$ if $n = 100$

We can also find the limit of a sum as n approaches ∞. [equal degree – ratio of leading coefficients]
Using the example above:

$$\lim_{n\to\infty}\sum_{i=1}^{n}\frac{i+1}{n^2}$$

ex. evaluate:

$$\lim_{n\to\infty}\sum_{i=1}^{n}\left(\frac{i}{n}\right)^2\left(\frac{1}{n}\right)$$

Practice Questions

Day 1: Worksheet & pg. 260/ # 1 – 14

Day 2: Pg. 260/# 15 – 20, 23, 24, 27, 28, 29 – 33