APCalc
Mr. C. Sdoutz

Date:________________

Unit 4: Integration
4.1 Indefinite Integration

If $f'(x) = 4x^3$, find $f(x)$.
*"undo" the differentiation process

$f(x) = x^4$ or $f(x) = x^4 + 1$ or $f(x) = x^4 - 2$ or…. There are infinite possibilities.

Definition
A function F is an *antiderivative* of f if $F'(x) = f(x)$.

A function can have an infinite number of antiderivatives because of all the different constants – called a "family" of solutions.

Finding antiderivatives is often referred to as solving a *differential equation* (DE).

ex. Find the general solution of the differential equation $f'(x) = 2x$.

$$\boxed{f(x) = x^2 + c}$$

Notation:

When solving a differential equation

$$\frac{dy}{dx} = f(x)$$

We rewrite

$$dy = f(x)dx$$

Then

$$y = \int f(x)\, dx = F(x) + c$$

integral sign integrand variable of integration constant of intregration

The process is called *antidifferentiation* or *indefinite integration*.

* indefinite integral = antiderivative

The variable of integration is important.

ex. $\int 2\, dx = 2x + c$ $\int 2\, dt = 2t + c$

Differentiation and indefinite integration are *inverse* operations.

Basic Integration Rules

1. $\int 0\,dx = c$
2. $\int k\,dx = kx + c$
3. $\int kf(x)\,dx = k\int f(x)\,dx$
4. $\int [f(x) \pm g(x)]\,dx = \int f(x)\,dx \pm \int g(x)\,dx$
5. $\int x^n\,dx = \frac{x^{n+1}}{n+1} + c;\, n \neq -1$

ex. Find $\int 3x\,dx$

$$= 3\int x\,dx$$

$$= 3\left(\frac{x^2}{2} + c\right)$$

$$\boxed{= \frac{3}{2}x^2 + c}$$

ex. Find $\int \frac{1}{x^3}\,dx$ *rewrite

$$= \int x^{-3}\,dx$$

$$= \frac{x^{-2}}{-2} + c$$

$$\boxed{= \frac{-1}{2x^2} + c}$$

ex. Find $\int 2sinx\,dx$

$$= 2\int sinx\,dx$$

$$= 2(-cosx + c)$$

$$\boxed{= -2cosx + c}$$

ex. Find $\int (x+2)\,dx$

$$= \int x\,dx + \int 2\,dx$$

$$= \frac{x^2}{2} + c_1 + 2x + c_2$$

$$\boxed{= \frac{x^2}{2} + 2x + c}$$

ex. Find $\int \frac{x^3+3}{x^2}\,dx$

*note: there is no product or quotient rule for antiderivatives

$$\neq \frac{\int x^3 + 3\,dx}{\int x^2\,dx}$$

$$= \int (x + 3x^{-2})\,dx$$

$$= \frac{x^2}{2} + \frac{3x^{-1}}{-1} + c$$

$$\boxed{= \frac{x^2}{2} - \frac{3}{x} + c}$$

If given an ***initial condition*** (a point) we can find a ***particular solution*** to the differential equation (ie. determine the value of c through substitution).

ex. Solve the D.E. $f'(x) = \frac{1}{x^2}$ if $f(1) = 0$

Step 1: find the general solution

$$f(x) = \int x^{-2}\,dx$$

$$= \frac{x^{-1}}{-1} + c$$

$$f(x) = \frac{-1}{x} + c$$

Step 2: solve for c using the initial condition

$$f(1) = 0$$

$$\frac{-1}{(1)} + c = 0$$

$$c = 1$$

Thus the particular solution is

$$f(x) = \frac{-1}{x} + 1$$

$$\boxed{f(x) = \frac{x-1}{x}}$$

Practice Questions

Pg. 248/# 5 – 16, 19, 21, 23 – 26, 29, 31, 32, 51 – 54, 56, 57, 63, [64], 70, 71, 73, [81 – 84]