APCalc Date:________________
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Unit 3: Applications of Differentiation
3.7 Optimization Problems

ex. The product of two positive whole numbers is 72. Find the numbers such that the sum of the first and twice the second is minimized.

Let x & y be the numbers.

$m = 2x + y$ the *primary equation* (the one being optimized)

$xy = 72$ the *secondary equation* (use to eliminate a variable in the primary)
$y = \frac{72}{x}$

$\therefore m = 2x + 72x^{-1}$ domain of x: $[1, 72]$

$$m' = 2 - \frac{72}{x^2} = 0$$
$$2 = \frac{72}{x^2}$$
$$x^2 = 36$$
$$x = \pm 6 \text{ but only } x = 6 \text{ is in the interval}$$

*closed interval on domain, so use absolute extrema approach (ie. Test the endpoints and the critical]

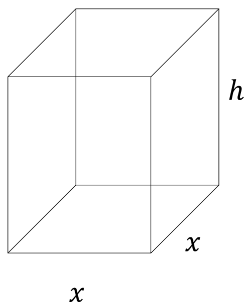
Test: $m(1) = 74$ $m(6) = 24$ $m(72) = 145$

Thus, minimum when $x = 6$. $y = \frac{72}{6} = 12$

The numbers are 6 & 12.

*note: The solution is very often at a critical and NOT at the endpoints of the interval, but officially we still have to try to determine the interval and test the endpoints.

ex. An open box (no lid) with a square base is to be constructed using 108 cm^2 of material. What dimensions will produce maximum volume for the box?



primary: $V = x^2 h$

secondary: $x^2 + 4xh = 108$

$$h = \frac{108 - x^2}{4x} = \frac{27}{x} - \frac{x}{4}$$

$$V = x^2\left(\frac{27}{x} - \frac{x}{4}\right)$$

$$V = 27x - \frac{x^3}{4}$$

Domain $(0, \sqrt{108})$

$$V' = 27 - \frac{3x^2}{4} = 0$$

$$27 = \frac{3x^2}{4}$$

$$3x^2 = 108$$

$$x^2 = 36$$

$x = \pm 6$ but only $x = 6$ in our interval

Open interval (can't test endpoints), use 1st or 2nd derivative test to verify that the critical IS a maximum:

$$V'' = -\frac{3x}{2}$$

$$V''(6) = -\frac{3(6)}{2} < 0$$

Thus, $x = 6$ is a max.

$$h = \frac{108 - 6^2}{4(6)} = 3$$

Thus, the max volume of 108 cm^3 is achieved with dimensions 6 cm × 6 cm × 3 cm

Practice Questions

Pg. 210/# 3, 6, 7, 9, 15, 16, 17c, [21, 28], 31, [37], 39