APCalc Date:_____
Mr. C. Sdoutz

Unit 3: Applications of Differentiation
3.5 Curve Sketching

ex. sketch $f(x) = \frac{2(x^2-9)}{x^2-4}$

x-intercepts (let $y = 0$): $\frac{2(x^2-9)}{x^2-4} = 0$

$x^2 - 9 = 0$

$x = \pm 3$

y-intercepts (let $x = 0$): $y = \frac{2(0^2-9)}{0^2-4}$

$= \frac{-18}{-4}$

$y = 4.5$

vertical asymptotes:

*discontinuities such that $f(c) = \frac{k}{0}$

$x = \pm 2$

horizontal asymptotes:

* $\lim_{x \to \pm\infty} f(x)$

$y = 2$

Criticals of f (possible extrema) & criticals of f' (possible inflection points)

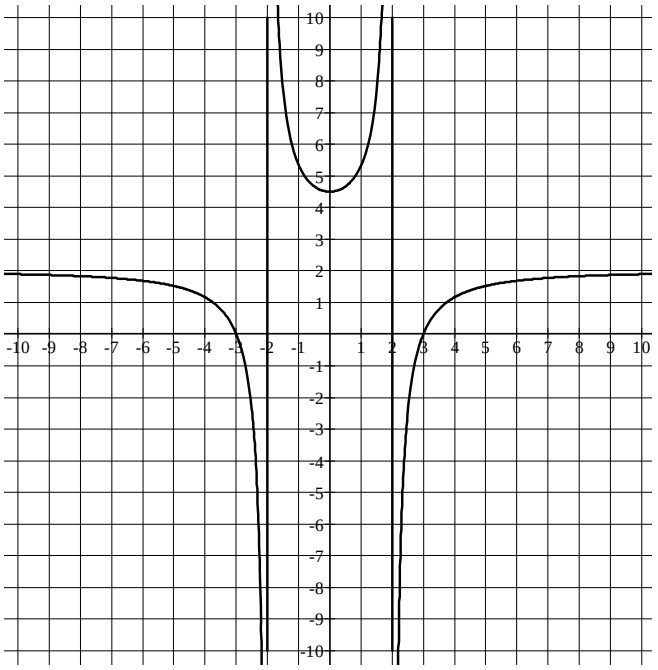
$f'(x) = \frac{20x}{(x^2-4)^2} = 0$ *undefined at $x = \pm 2$ but NOT criticals as $f(\pm 2)$ undefined (vertical asymptotes)

critical: $x = 0$

$f''(x) = \frac{-20(3x^2+4)}{(x^2-4)^3}$ *no criticals

Intervals & criticals/ undefined	$f(x)$	$f'(x)$	$f''(x)$	Characteristic
$(-\infty, -2)$		$-$	$-$	Dec, concave down
-2	∞	∞	∞	Vert. asymptote
$(-2, 0)$		$-$	$+$	Dec, concave up
0	4.5	0	$+$	Relative min
$(0, 2)$		$+$	$+$	Inc, concave up
2	∞	∞	∞	Vert. asymptote
$(2, \infty)$		$+$	$-$	Inc, concave down

*note: you can also generate points wherever uncertain



ex. sketch $f(x) = x^5 - 4x^3$

x-intercepts (let $y = 0$): $x^5 - 4x^3 = 0$

$x^3(x^2 - 4) = 0$

$x = 0, \pm 2$

y-intercepts (let $x = 0$): $y = (0)^5 - 4(0)^3$

$y = 0$

vertical asymptotes: none (polynomial)

horizontal asymptotes: none (polynomial)

Criticals of f (possible extrema) & criticals of f' (possible inflection points)

$f'(x) = 5x^4 - 12x^2 = 0$

$x^2(5x^2 - 12) = 0$

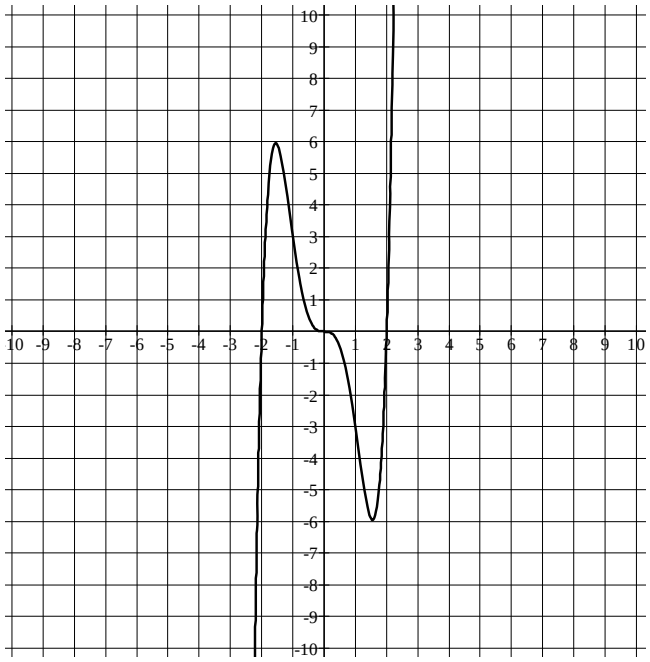
criticals: $x = 0$ or $x = \pm\sqrt{\frac{12}{5}} \cong \pm 1.5$

$f''(x) = 20x^3 - 24x = 0$

$4x(5x^2 - 6) = 0$

criticals: $x = 0$ or $x = \pm\sqrt{\frac{6}{5}} \cong \pm 1.1$

Intervals & criticals/ undefined	$f(x)$	$f'(x)$	$f''(x)$	Characteristic
$(-\infty, -1.5)$		$+$	$-$	Inc, concave down
-1.5	5.9	0	$-$	Relative max
$(-1.5, -1.1)$		$-$	$-$	Dec, concave down
-1.1	3.7		0	Inflection pt
$(-1.1, 0)$		$-$	$+$	Dec, concave up
0	0	0	0	Inflection pt
$(0, 1.1)$		$-$	$-$	Dec, concave down
1.1	-3.7		0	Inflection pt
$(1.1, 1.5)$		$-$	$+$	Dec, concave up
1.5	-5.9	0	$+$	Relative min
$(1.5, \infty)$		$+$	$+$	Inc, concave up



Practice Questions

Pg. 202/ # 5, 9, 13, 17, 29, 31 – 33