Calculus12 Date:________________
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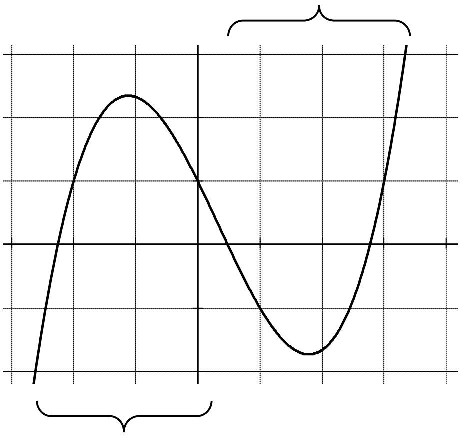
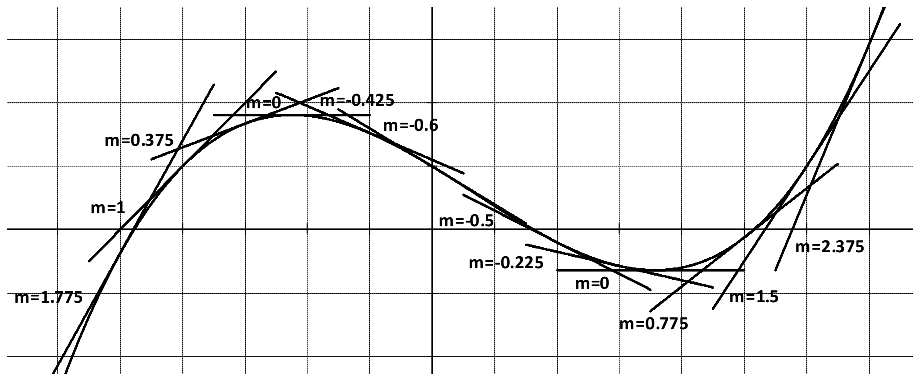
<u>Unit 3: Applications of Differentiation</u>
<u>3.4 Concavity & The 2nd Derivative Test</u>

Concavity – refers to whether the graph of a function is curving upward (concave up) or curving downward (concave down).

Concave up



Concave down



Tracking the values of various tangent slopes as x gets larger demonstrates that the slope values (ie. the derivative values) get smaller while the graph is concave down and get larger while the graph is concave up.

*applet

<u>Definition of Concavity</u>
Let $f(x)$ be a differentiable function on an interval I.
The graph of $f(x)$ is *<u>concave upward</u>* on I if $f'(x)$ is ____________ on the interval.
The graph of $f(x)$ is *<u>concave downward</u>* on I if $f'(x)$ is ____________ on the interval.

From 3.3 we know that a function is increasing when its derivative is _________ and is decreasing when its derivative is __________.

Thus, if $f'(x)$ is increasing then its derivative $f''(x)$________

Similarly, if $f'(x)$ is decreasing then its derivative $f''(x)$________

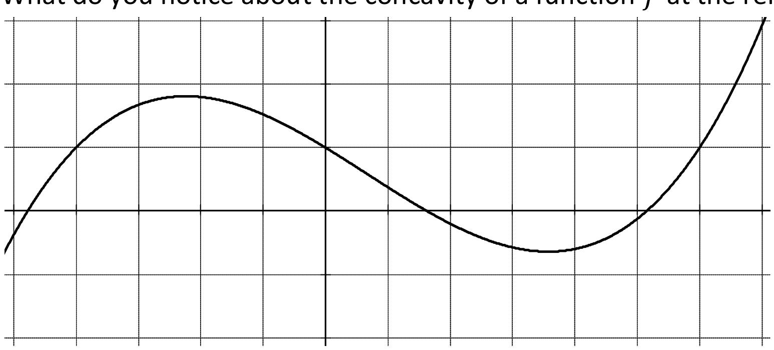
<u>Test for Concavity</u>
Let $f(x)$ be a twice differentiable function on an interval I.
If $f''(x) > 0$ for all x in I then the graph of $f(x)$ is concave upward on I.
If $f''(x) < 0$ for all x in I then the graph of $f(x)$ is concave downward on I.

ex. Determine the open intervals on which the graph of $f(x) = \frac{x^3}{3} - x$ is concave up or down.
*note – to solve this question we will determine the intervals on which $f'(x)$ is increasing or decreasing

The point where concavity changes is called a *point of inflection*. In the previous example the point of inflection was _______

Points of inflection possibly occur at criticals of $f'(x)$. [ie. where the derivative values reach a relative minimum or maximum]

What do you notice about the concavity of a function f at the relative extrema of f?



At a relative maximum the graph is

At a relative minimum the graph is

<u>The 2nd Derivative Test for Extrema</u>

Let f be a function with a critical value c and a 2nd derivative $f''(x)$ which exists on an interval containing c.

If $f''(c) > 0$ then $f(c)$ is a relative minimum.

If $f''(c) < 0$ then $f(c)$ is a relative maximum.

*If $f''(c) = 0$ then the test fails. Most likely this is because $f(c)$ is an inflection point, but we use the 1st derivative test to double check for any more extrema at $f(c)$.

ex. Use the 2nd derivative test to identify the relative extrema of $f(x) = -3x^5 + 5x^3$

Practice Questions

Pg. 184/ # 1 – 19, 21 – 31, 35, 47 – 50, 58, 63, [77 – 80]

*bracketed questions are more challenging