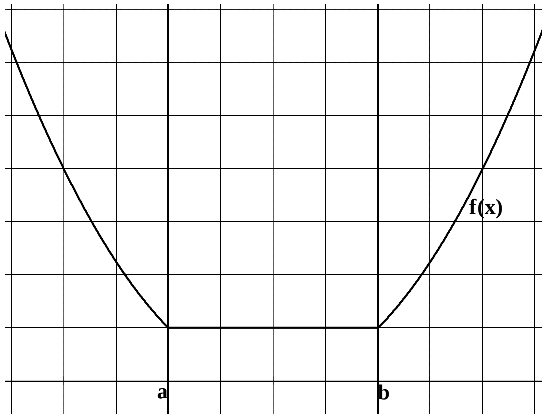
Unit 3: Applications of Differentiation
3.3 Increasing & Decreasing Functions



Decreasing on $(-\infty, a)$: as x moves right $f(x)$ gets smaller – slopes are negative.

Increasing on (b, ∞): as x moves right $f(x)$ gets larger – slopes are positive.

Constant on (a, b): as x moves right $f(x)$ is the same – slope is zero.

* remember the 1st derivative tell us the slope of the function, thus use the 1st derivative to test

1. If $f'(x) > 0$ for all x in (a, b) then $f(x)$ is *increasing* on (a, b).
2. If $f'(x) < 0$ for all x in (a, b) then $f(x)$ is *decreasing* on (a, b).
3. If $f'(x) = 0$ for all x in (a, b) then $f(x)$ is *constant* on (a, b).

ex. Determine the open intervals on which $f(x) = x^3 - \frac{3}{2}x^2$ is increasing or decreasing.

Step 1: Determine the critical values ($f'(x) = 0$ or undefined)

$f'(x) = 3x^2 - 3x$ no undefined critical

$3x^2 - 3x = 0$

$3x(x - 1) = 0$

$x = 0$ or $x = 1$

Step 2: Set up intervals with the critical values to split the domain of all real numbers. Choose a test value from each interval and determine if $f'(x)$ is $+$ or – at that value. Conclude which intervals are increasing ($f'(x) > 0$) and which are decreasing ($f'(x) < 0$).

	$(-\infty, 0)$	$(0, 1)$	$(1, \infty)$
test:	$x = -1$	$x = 0.5$	$x = 2$
	$f'(-1) > 0$	$f'(0.5) < 0$	$f'(2) > 0$
	increasing	decreasing	increasing

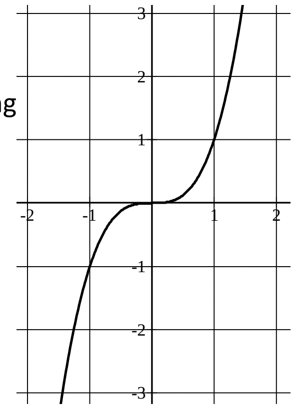
Thus, $f(x) = x^3 - \frac{3}{2}x^2$ is increasing on $(-\infty, 0)$ & $(1, \infty)$, and is decreasing on $(0, 1)$.
[check with graphing calculator]

*note: inc/dec will often alternate on subsequent intervals but **NOT** always

A function is ***strictly monotonic*** if it is always increasing, always decreasing, or always constant.

ex. $f(x) = x^3$

*always increasing



We can use the increasing/decreasing nature around a critical point to determine whether a critical point is a relative maximum, relative minimum, or neither.

Going back to our first example: $f(x) = x^3 - \frac{3}{2}x^2$

	$(-\infty, 0)$	$(0, 1)$	$(1, \infty)$
test:	$x = -1$	$x = 0.5$	$x = 2$
	$f'(-1) > 0$	$f'(0.5) < 0$	$f'(2) > 0$
	increasing	decreasing	increasing

As x approaches 0 $f(x)$ is increasing but then it decreases afterwards, thus $x = 0$ must produce a relative maximum.
Thus, $f(x)$ has a ***relative maximum*** at $(0, 0)$because $f'(0) = 0$ and $f'(x)$ switches from positive to negative at $x = 0$.

As x approaches 1 $f(x)$ is decreasing but then it increases afterwards, thus $x = 1$ must produce a relative minimum.
Thus, $f(x)$ has a ***relative minimum*** at $\left(1, -\frac{1}{2}\right)$ because $f'(1) = 0$ and $f'(x)$ switches from negative to positive at $x = 1$.

The First Derivative Test for Relative Extrema

Let $f(x)$ be a differentiable function on an interval containing a critical value c, then:

1. If $f'(x)$ changes from negative to positive (ie. $f(x)$ switches from decreasing to increasing) at c then $f(c)$ is a relative minimum of $f(x)$.
2. If $f'(x)$ changes from positive to negative (ie. $f(x)$ switches from increasing to decreasing) at c then $f(c)$ is a relative maximum of $f(x)$.

ex. Identify the relative extrema of $f(x) = (x^2 - 4)^{2/3}$

$$f'(x) = \frac{2}{3}(x^2 - 4)^{-1/3}(2x)$$
$$= \frac{4x}{3(x^2 - 4)^{1/3}}$$

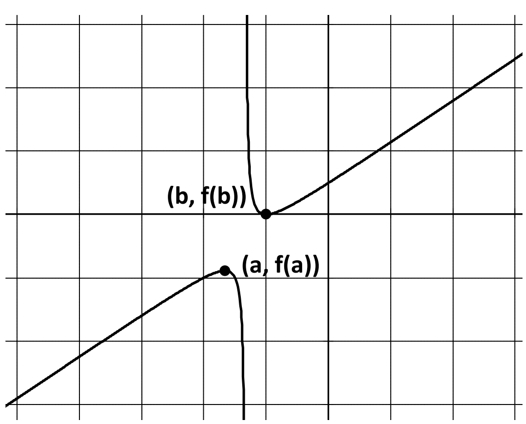
∴ critical values at $x = \pm 2$ since $f'(\pm 2)$ is undefined and a critical value at $x = 0$ since $f'(0) = 0$

$(-\infty, -2)$	$(-2, 0)$	$(0, 2)$	$(2, \infty)$
$f'(-3) < 0$	$f'(-1) > 0$	$f'(1) < 0$	$f'(3) > 0$
decreasing	increasing	decreasing	increasing

Relative minimums at $(-2, 0)$ & $(2, 0)$ because $f'(\pm 2)$ is undefined and $f'(x)$ switches from negative to positive at $x = \pm 2$.

Relative maximum at $\left(0, \sqrt[3]{16}\right)$ because $f'(0) = 0$ and $f'(x)$ switches from positive to negative at $x = 0$.

Note: Do NOT test for relative extrema like absolute extrema in 3.1 (evaluating $f(x)$ to see which the smallest/largest output is). Just because one value is smaller than another does not mean the smaller is a relative minimum and the larger a relative maximum.



ex.

The function has critical values at a & b.

$f(a) < f(b)$ but $f(a)$ is relative maximum and $f(b)$ is a relative minimum

Practice Questions

Pg. 176/ # 1 – 16, 19 – 28, [37 – 42], 49, 55 – 57, [65 – 68]

*bracketed questions are more challenging