APCalc
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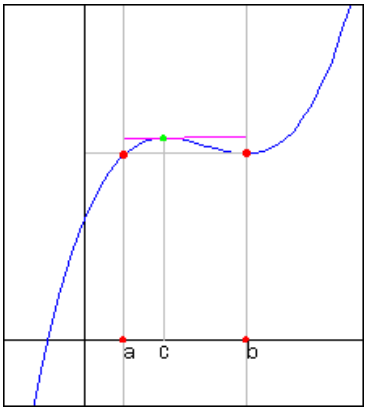
Date:_____

Unit 3: Applications of Differentiation
3.2 Rolle's Theorem & The Mean Value Theorem

Rolle's Theorem

Let f be continuous on $[a, b]$ and differentiable on (a, b). If $f(a) = f(b)$ then there exists at least one number c in (a, b) such that $f'(c) = 0$.

ie. If a function starts and stops at the same vertical value then there must be a horizontal tangent line somewhere between.



[applet]

ex. Identify the x-intercepts of $f(x) = x^2 - 3x + 2$ and show that $f'(x) = 0$ at some point between these zeros.

$$x^2 - 3x + 2 = 0$$
$$(x - 2)(x - 1) = 0$$
$$x = 1 \ \ or \ \ x = 2$$

$f(x)$ is differentiable and continuous everywhere as it is a polynomial function and $f(1) = f(2) = 0$, thus by Rolle's Theorem there exists a value c in $(1, 2)$ such that $f'(c) = 0$.

To find c:

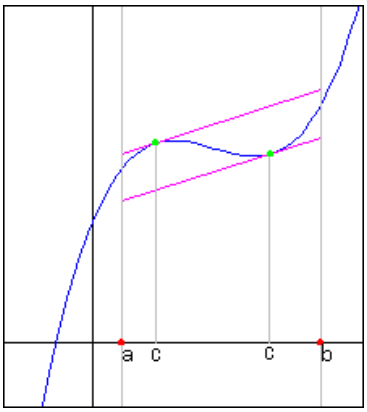
$$f'(c) = 2c - 3 = 0$$
$$c = \frac{3}{2}$$

The Mean Value Theorem

If f be continuous on $[a, b]$ and differentiable on (a, b) then there exists a number c in (a, b) such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

ie. At some point in the interval the instantaneous rate of change will equal the average rate of change



[applet]

ex. Two police cars with radar guns are positioned 5 km apart on a highway with a 60 km/h speed limit. A truck passes the first officer with a speed of 55 km/h and then passes the second officer four minutes later with a speed of 50 km/h. The office then pulls over the truck and gives the driver a speeding ticket. How does he justify the ticket?

The car has some position function $s(t)$ which we will assume to be continuous and differentiable.
Let $s(0) = 0$ be the when the first police car clocks the truck.
$4\ min = \frac{1}{15}\ hr$; thus $s\left(\frac{1}{15}\right) = 5$

$$Average\ velocity = \frac{s\left(\frac{1}{15}\right) - s(0)}{\frac{1}{15} - 0}$$
$$= \frac{5}{\frac{1}{15}} = 75\ km/h$$

By the mean value theorem there must exist a time c in $\left(0, \frac{1}{15}\right)$ such that $s'(c) = v(c) = 75\ km/h$.
Thus, the car was speeding at least once between the two police cars.

Practice Questions

Pg. 167/ # 1, 3 – 11, 21, 23, 27 – 33, 41, 43, 45, 47 – 50