

Unit 3: Applications of Differentiation
3.1 Extrema on an Interval

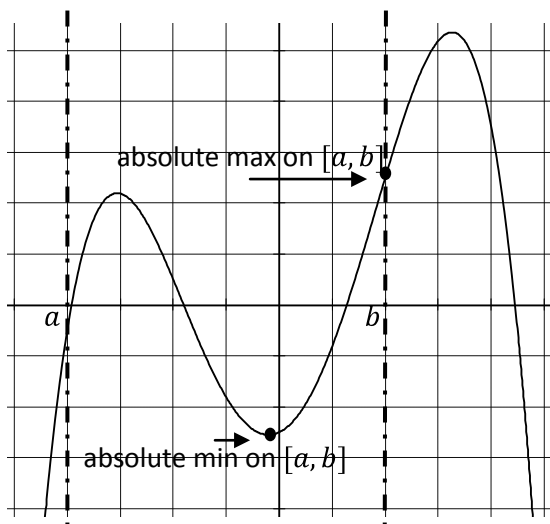
Definition of Extrema

Let a function f be defined on an interval I containing a value c

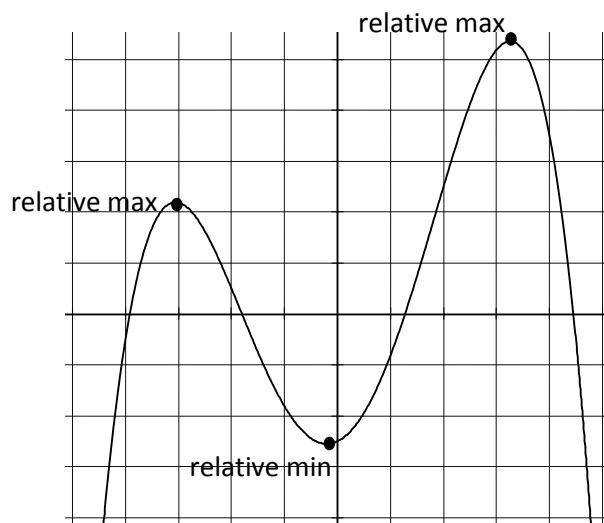
1. $f(c)$ is the minimum of f on I if $f(c) \leq f(x)$ for all x in I
ie. no other value in I produces a value smaller than $f(c)$
2. $f(c)$ is the maximum of f on I if $f(c) \geq f(x)$ for all x in I
ie. no other value in I produces a value larger than $f(c)$

Minimums and maximums are called extreme values or extrema.

ex. Extrema on a closed interval:



Extrema without interval:



Extreme Value Theorem

If f is continuous on a closed interval $[a, b]$ then f has both a minimum and maximum on the interval.

Definition of a critical

Let f be defined at c . If $f'(c) = 0$ or if f' is undefined at c , then c is a critical number (value) of f .

Relative extrema will occur at critical numbers.

Absolute extrema (on an interval) will occur at critical numbers or at the endpoints of the interval.

Finding extrema on a closed interval $[a, b]$

1. Identify critical numbers of f in (a, b)
2. Evaluate f at the critical
3. Evaluate the endpoint values $f(a)$ and $f(b)$
4. The least value is the min; the greatest value is the max

ex. Identify the extrema of $f(x) = 3x^4 - 4x^3$ on $[-1, 2]$.

$$f'(x) = 12x^3 - 12x^2$$

Defined for all x , thus set equal zero to identify criticals.

$$12x^3 - 12x^2 = 0$$

$$12x^2(x - 1) = 0$$

$$x = 0 \text{ or } x = 1$$

Evaluate function at critical values:

$$f(0) = 0 \quad f(1) = -1$$

Evaluate function at interval endpoints:

$$f(-1) = 7 \quad f(2) = 16$$

$\therefore$ minimum value is -1 at the point $(1, -1)$ & maximum value is 16 at the point $(2, 16)$

ex. Identify the extrema of $f(x) = \sqrt[3]{x}$ on $[-1, 1]$.

$$f'(x) = \frac{1}{3}x^{-2/3} = \frac{1}{3\sqrt[3]{x^2}}$$

Never equal to 0, but undefined at critical $x = 0$.

$$f(0) = 0 \quad f(-1) = -1 \quad f(1) = 1$$

$\therefore$ minimum value is -1 at the point $(-1, -1)$ & maximum value is 1 at the point $(1, 1)$

Practice Questions

Pg. 160/ # 7 – 25 odd, 29 – 33, 35, 48, 53 – 56